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Effect of Pasternak foundation: Structural modal identification for vibration of FG shell

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Abstract. Employment of the wave propagation approach with the combination of Pasternak foundation equation gives birth to the shell frequency equation. Mathematically, the integral form of the Lagrange energy functional is converted into a set of three partial differential equations. A cylindrical shell is placed on the elastic foundation of Pasternak. For isotropic materials, the physical properties are same everywhere, whereas the laminated and functionally graded materials, they vary from point to point. Here the shell material has been taken as functionally graded material. The influence of the elastic foundation, wave number, length and height-to-radius ratios is investigated with different boundary conditions. The frequencies of length-to-radius and height-to-radius ratio are counter part of each other. The frequency first increases and gain maximum value in the midway of the shell length and then lowers down for the variations of wave number. It is found that due to inducting the elastic foundation of Pasternak, the frequencies increases. It is also exhibited that the effect of frequencies is investigated by varying the surfaces with stainless steel and nickel as a constituent material. MATLAB software is utilized for the vibration of functionally graded cylindrical shell with elastic foundation of Pasternak and the results are verified with the open literature.

Keywords: elastic foundation; FGM; MATLAB; Nickel; wave number

1. Introduction

Study of vibration characteristics of cylindrical shells is a widely area of research in applied mathematics and theoretical mechanics. Analytical investigation of vibrations of these shell are performed to estimate the probable dynamical response. Variations in the shell physical parameters are inducted to enhance their strength and stability. Vibration of shell problems occur in industrial engineering fields. Their vibration analysis predicts to approximate their experimental results. More the shell material sustains a load due to physical situations, the more the shell is stable. Any predicted fatigue due to burden of vibrations is evaded by estimating their dynamical aspects. Addition of more physical parameters may give rise more instability in a system of a submerged cylindrical shell (CSs). During the recent years, study of cylindrical shell with elastic foundations has gained the attention of researchers doing work on their vibration characteristics. Advanced composite materials keep extreme particular stiffness, strength and are resistant to corrosion. The elastic foundation equation is applied to influence on the shell vibrations. Firstly, Love (1888) presented the Kirchhoff's hypotheses for plates. After that this theory became a foundation stage for building new ones by changing physical terms expressions. More than one type of materials is used to structure the functionally graded (FG) materials and their physical properties vary from one surface to the other surface. In these surfaces, one has highly heat resistance property while other may preserve great dynamical perseverance and differs mechanically and physically in regular manner from one surface to other surface, making them of dual physical appearance.

All these materials have changeable outer and inner sides and their physical properties greatly differ from each other (Suresh and Mortensen 1997, Koizumi 1997). These materials are organized by various techniques and their applications are seen in dynamical elements such as plates, beams and shells. Moreover, these materials are also observed in space crafts, nuclear reactors and missiles technology etc.

Loy and Lam (1997) investigated shell vibrations with ring supports that restricted the motion of cylindrical shells in the transverse direction. This influence was inducted by the polynomial functions. Ahmed *et al.* (2019) concerned with post-buckling investigation of nano-scaled beams constructed from porous functionally graded (FG) materials taking into account geometrical imperfection shape. Hence, two types of nanobeams which are perfect and imperfect have been studied. Porous FG materials are classified based on even or uneven porosity distributions. Xiang *et al.* (2002) formed some closed form solution functions for studying vibrations of cylindrical shells. The mid-way ring supports were clamped around the shells. Faleh *et al.* (2020)

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investigated the pulse load effects on forced transient vibrations of porous crystalline shells. A crystalline material contains many voids inside it and also there are nano-size grains which define the material character. The formulation for crystalline cylindrical shell is provided by first-order shell theory and a numerical approach is used to solve shell equations. Sewall and Naumann (1968) considered the vibration analysis of CSs based on analytical and experimental methods. The shells were strengthened with longitudinal stiffeners. Fenjan *et al.* (2019) studied the free vibrational characteristics of porous steel double-coupled nanoplate system in thermo- elastic medium. Different pore dispersions called uniform, symmetric and asymmetric have been defined. Nonlocal strain gradient theory (NSGT) containing two scale parameters have been adopted to establish size-dependent modeling of the system. Sharma (1974) analyzed vibration frequencies circular cylinder with using the Rayleigh-Ritz formulation and made comparisons of his results with some experimental ones. Mehar *et al.* (2017a, b, c, d) studied the frequency response of FG CNT and reinforced CNT using the simple deformation theory, finite element modeling, Mori-Tanaka scheme. They investigated a new frequency phenomena with the combination of Lagrange strain, Green-Lagrange, for double curved and curved panel of FG and reinforced FG CNT. The characteristics of sandwich and grades CNT was found with labeling the temperarure environ. The thermoelastic frequency of single shallow panel was determined using Mori-Tanka formulations. The research of these authors has opened a new frequency spectrum for other material researchers. Mehar and Panda (2018b) investigated the curved shell and CNT vibration with thermal environment using higher order deformation theory. These CNT was mixed with different configurations of the layers. The results have been verified with the earlier investigations. Fenjan *et al*. (2019) studied the dynamic stability of a porous metal foam nano-dimension plate on elastic substrate exposed to bi-axial time-dependent forces. Various pore contents based on uniform and non-uniform models have been introduced. Chung *et al.* (1981) investigated the vibrations of fluid-filled CSs and presented an analysis of experimental and analytical investigation.

Goncalves and Batista (1987) gave an analytical investigation of CSs partially filled and submerged in a fluid. Mehar and Panda (2016a, b, 2018a) computed the vibration behavior, bending and dynamic response of FG reinforced CNT using shear deformation theory and finite element method. For the sake of generality, the mathematical model was presented with the mixture of Green Lagrange method. The convergence of these methodologies has been checked for the variety of results. The composite plates with different graded was investigated with isotropic and core phase. Jiang and Olson (1994) recommended the characteristics of analysis of stiffened shell using finite element method to diminish large computational efforts which are required in the conventional finite element analysis. Salah *et al.* (2019) employed a simple four-variable integral plate theory for examining the thermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Wang *et al.* (1997) scrutinized the vibrations of ring-stiffened CSs using Ritz polynomial functions. Materials of both shells and rings were of isotropic nature. These shells were stiffened with isotropic rings having three types of locations on the shell outer surface. To increase the stiffness of CSs was stabilized by ring-stiffeners. Isotopic materials are the constituents of these rings. Mehar and Panda (2018c) investigated numerically the deflection behavior of carbon nanotube-reinforced composite plate using the finite-element method and the result accuracy is established via three-point experimental bending test data. Batou *et al.* (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. A large use of shell structures in practical applications makes their theoretical analysis an important field of structural dynamics. Since a shell problem is a physical one, so their vibrational behaviors are distorted by variations of physical and material parameters. To elude any complications which may risk a physical system their analytical investigation was done. Sharma *et al.* (1998) determined frequencies of composite cylindrical shells containing fluid. They estimated the axial modal deformations by trigonometric functions. Mehar and Panda (2018d) developed a general mathematical model for the evaluation of the theoretical flexural responses of the functionally graded carbon nanotube- reinforced composite doubly curved shell panel using higher-order shear deformation theory with thermal load. Ergin and Temarel (2002) did a vibration study of cylindrical shells. The shells lied in a horizontal direction and contained fluid and submerged in it. Mehar and Panda, (2018e) reported the nonlinear finite solutions of the nonlinear flexural strength and stress behaviour of nano sandwich graded structural shell panel under the combined thermomechanical loading. Najafizadeh and Isvandzibaei (2007) applied ring supports to CSs for vibration analysis of along the tangential direction and founded their research on angular deformation theory of higher order. The angular deformation was used for shell equations and determined the effects of constituent volume fractions and shell configurations on the shell vibrations. FG material parameters were changed step by step. Mehar *et al.* (2018a) evaluated the frequency behavior of nanolpate structure using FEM including the nonlocal theory of elasticity. Computer generated results are created by using the software first time robustly to check the vibration of nanoplate. The efficiency was checked by comparing the results of available data. Shah *et al.* (2009) and Sofiyev and Avcar (2010) studied stability of CSs based on Rayleigh-Ritz and Galerkin technique using elastic foundations. The structures of cylindrical shell are tackled under the exponential law and axial load. Naeem *et al.* (2013) conducted the vibrational behavior of submerged FG-CSs. The problem of submerged cylindrical shells were frequently met where fluid envelopes a structure. The present problem consists of a CSs submerged in a fluid and surrounded by ring supports. Ansari *et al.* (2015) performed nonlocal model for the frequencies of multi-walled carbon nanotubes with small effects subject to various boundary conditions (BCs) using Rayleigh-Ritz technique. The governing equation was formulated based on Flügge's and nonlocal shell theory. Some new resonant frequencies were identified with the association of vibrational modes and circumferential modes into shell model. Mehar *et al.* (2018b) studied the bending responses of nanotube-reinforced curved sandwich shell panel structure under the influence of the thermomechanical loading. Further, the temperature dependent material properties of the sandwich structure are assumed to evaluate the exact responses. Pankaj *et al.* (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Recently some researcher used different methods for nonlinear modeling (Avcar 2019, Karami *et al.* 2017, 2018, Madani *et al.* 2016, Simsek 2011, Yeh 2016, Bouhlali *et al.* 2019, Ayat *et al.* 2018) and deformation theory (Mehar *et al.* 2016), finite element method (Mehar *et al.* 2018c), Green-Lagrange strain field (Mehar *et al.* 2018c), and multiscale modeling approach (Mehar and Panda 2019, Mehar *et al.* 2019).

According to our knowledge, up to now little is known about the vibration analyses of varying two different materials and has not been investigated for FG-CS with elastic foundation of Pasternak based on wave propagation approach. It is also exhibited that the effect of frequencies by varying the stainless steel and nickel as a constituent material. Also the Love shell model based on the wave propagation approach for estimating fundamental natural frequency has been developed to converge more quickly than other methods and models. The influence of the Pasternak foundation is investigated with different edge conditions. The frequency behavior is investigated versus circumferential wave number, length-to-radius and height-to-radius ratio. Moreover, frequency pattern is found for the various values of Pasternak foundations. The frequency first increases and gain maximum value in the midway of the shell length and then lowers down for wave number. The presented vibration modeling and analysis of CSs may be helpful especially in applications such as oscillators and in non-destructive testing. To elude any complications which may risk a physical system their analytical investigation is done.

2. Functionally graded materials

On mixing two or more than two materials like ceramic and metal, functionally graded materials are obtained. This type of material are working in high-temperature dependence material goods. So the Young's modulus E_{fgm}, Poisson ratio v_{fgm} and mass density ρ_{fgm} are defined as

$$E_{fgm} = (E_1 - E_2)\left(\frac{z}{h} + 0.5\right)^N + E_2 \tag{1}$$

$$\rho_{fgm} = (\rho_1 - \rho_2)\left(\frac{z}{h} + 0.5\right)^N + \rho_2 \tag{2}$$

$$v_{fgm} = (v_1 - v_2)\left(\frac{z}{h} + 0.5\right)^N + v_2 \tag{3}$$

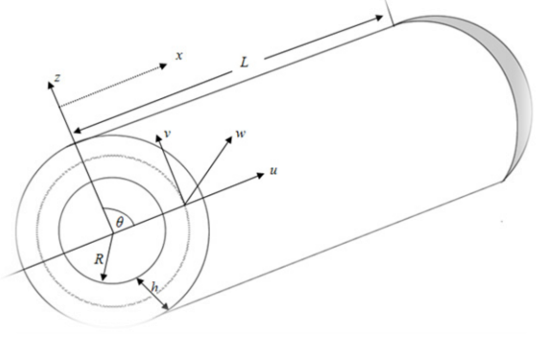

Fig. 1 Geometry of CS

where N known as power law index and thickness and z is the coordinate which varies from zero to infinity.

The distribution of volume fraction for all types of CSs are assumed as

$$V_f = \left[\frac{z}{h} + \frac{1}{2}\right]^N \tag{4}$$

3. Volume fraction law

This supposition simplifies the procedure of evaluation of integrals denoting material stiffness moduli. A FG-CS consisting of two constituent materials. In these categories Nickel and Stainless steel are used as the interior surfaces and the exterior surface respectively, but their arrangement has profound influence on the formation of FG-CSs. The order of the FG constituent materials is reversed as Type-I and Type-II. The volume fraction V_f are designated for CSs, respectively. Their exist a property of unity for composite material.

$$V_f = \left(z/h + 0.5\right)^N \tag{5}$$

The material parameters: E_1, E_2, v_1, v_2, and ρ_1, ρ_2 for constituents materials Stainless steel and Nickel at a temperature of 300K. Toulokian *et al.* (1967) stated the material properties σ at high temperature environ, with temperature-dependents which is a function of temperature. In Eq. (6), the constants (σ_0, σ_{-1}, σ_1, σ_2, σ_3) are different for different material.

$$\sigma = \sigma_0\left(\sigma_{-1}T^{-1} + \sigma_1 T + \sigma_2 T^2 + \sigma_3 T^3\right) \tag{6}$$

At temperature 300K, for stainless steel and nickel, the material properties for FG-CS are: E, υ, ρ for Stainless steel are 2.07788×10^{11} N/m^3, 0.317756 and 8166 Kg/m^3 and Nickel are 2.05098×1011 N/m^2, 0.3100, and 8900 Kg/m^3.

4. Theoretical formation

The shell is assumed to have length L, thickness h and the radius R for cylindrical shell with its coordinate system (x, θ, z) as shown in Fig. 1. The x, θ co-ordinate are assumed to be along longitudinal and circumferential direction, respectively and z-co-ordinates are taken in its radial directions. The equation of motion of cylindrical shell (See Fig. 1) are expressed ass in the form of differential operator as

$$\tau_{11}u+\tau_{12}v+\tau_{13}w=\rho_t\frac{\partial^2 u}{\partial t^2} \tag{7a}$$

$$\tau_{21}u+\tau_{22}v+\tau_{23}w=\rho_t\frac{\partial^2 v}{\partial t^2} \tag{7b}$$

$$\tau_{31}u+\tau_{32}v+\tau_{33}w=\rho_t\frac{\partial^2 u}{\partial t^2}-G\nabla^2 w \tag{7c}$$

Where G represents the 'Pasternak' elastic foundation and the differential operator ∇^2 is expressed as

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} \tag{8}$$

Hence

$$\tau_{11}=A_{11}\frac{\partial^2}{\partial x^2}+\left(\frac{A_{66}}{R^2}+\rho_t\right)\frac{\partial^2}{\partial\theta^2}$$

$$\tau_{12}=\frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x\partial\theta}+\frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^2}{\partial x\partial\theta}$$

$$\tau_{13}=\left(\frac{A_{12}}{R}-\rho_t R\right)\frac{\partial}{\partial x}-B_{11}\frac{\partial^3}{\partial x^3}-\frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^3}{\partial x\partial\theta^2}$$

$$\tau_{21}=\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+B_{66}}{R^2}+\rho_t R\right)\frac{\partial^2}{\partial x\partial\theta}$$

$$\tau_{22}=\left(A_{66}+\frac{3B_{66}}{R}+\frac{2D_{66}}{R^2}\right)\frac{\partial^2}{\partial x^2}+\left(\frac{A_{22}}{R^2}+\frac{2B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^2}{\partial\theta^2}+\rho_t$$

$$\tau_{23}=\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)\frac{\partial}{\partial\theta}-\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial\theta^3}-\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+2D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2\partial\theta}-2\rho_t\frac{\partial}{\partial t}$$

$$\tau_{31}=-\frac{A_{12}}{R}\frac{\partial}{\partial x}+B_{11}\frac{\partial^3}{\partial x^3}+\left(\frac{B_{12}+2B_{66}}{R^2}\right)\frac{\partial^3}{\partial x\partial\theta^2}$$

$$\tau_{32}=-\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}+\rho_t\right)\frac{\partial}{\partial\theta}+\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial\theta^3}+\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2\partial\theta}+2\rho_t\frac{\partial}{\partial t}$$

$$\tau_{33}=-\frac{A_{22}}{R^2}+\rho_t+\frac{2B_{12}}{R}\frac{\partial^2}{\partial x^2}+\left(\frac{2B_{22}}{R^3}+\rho_t\right)\frac{\partial^2}{\partial\theta^2}-D_{11}\frac{\partial^4}{\partial x^4}-2\left(\frac{D_{12}+2D_{66}}{R^2}\right)\frac{\partial^4}{\partial x^2\partial\theta^2}-\frac{D_{22}}{R^4}\frac{\partial^4}{\partial\theta^4}$$

Many material researchers have used various methods to solve PDEs. Closed form functions exist for solutions of shell motion equations for some types of edge conditions. For rest of boundary conditions, the numerical solutions are obtained by approximate methods. Wave propagation method is made to extract approximate solution of shell controlling equations because these techniques provide results robustly with sufficient accuracy. This method is appropriate, straightforward to extract the shell vibration frequencies. Differential equations are generated involving the three dependent variables. Boundary conditions existing at the shell ends are met by these functions. For solving the shell problem, first the special variables x, θ and temporal variable t are split. For this purpose the following modal displacement expressions for the deformation functions: u, v and w are presupposed as

$$u(x,\theta,t)=U\,e^{-ik_m x}\,cos(n\theta+\omega t) \tag{9a}$$

$$v(x,\theta,t)=V\,e^{-ik_m x}\,sin(n\theta+\omega t) \tag{9b}$$

$$w(x,\theta,t)=W\,e^{-ik_m x}\,cos(n\theta+\omega t) \tag{9c}$$

Where the parameters U, V and W represents the vibration amplitudes in the x, θ and z directions, correspondingly. k_m signifies the unknown axial deformation function that fulfills the end conditions stated at two shell ends. Substituting the values from τ_{11} to τ_{33} into corresponding Eqs. (9a), (9b), (9c), the frequency expression is framed in the form of Eigen value problem and three dynamical shell equations convert the following expressions

$$\left\{-k_m^2A_{11}-n^2\left(\frac{A_{66}}{R^2}+\rho_t\right)+\rho_t\omega^2\right\}U+\left\{-ink_m\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+2B_{66}}{R^2}\right)\right\}V+\left\{-ik_m^3B_{11}-ik_m\left(\frac{A_{12}}{R}-\rho_t R\right)-in^2k_m\left(\frac{B_{12}+2B_{66}}{R^2}\right)\right\}W=0 \tag{10a}$$

$$\left\{ink_m\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+B_{66}}{R^2}+\rho_t R\right)\right\}U+\left\{-k_m^2\left(A_{66}+\frac{3B_{66}}{R}+\frac{2D_{66}}{R^2}\right)-n^2\left(\frac{A_{22}}{R^2}+\frac{2B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)+\rho_t\,\rho_t\omega^2\right\}V+\left\{-n\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)-n^3\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)-nk_m^2\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+2D_{66}}{R^2}\right)+2\rho_t\omega\right\}W=0 \tag{10a}$$

$$\left\{ik_m\frac{A_{12}}{R}+ik_m^3B_{11}+in^2k_m\left(\frac{B_{12}+2B_{66}}{R^2}\right)\right\}+\left\{-n\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}+\rho_t\right)-n^3\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)-nk_m^2\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)+2\rho_t\omega\right\}V+\left\{-\frac{A_{22}}{R^2}+\rho_t-k_m^2\frac{2B_{12}}{R}-n^2\left(\frac{2B_{22}}{R^3}+\rho_t\right)-k_m^4D_{11}-2n^2k_m^2\left(\frac{D_{12}+2D_{66}}{R^2}\right)-n^4\frac{D_{22}}{R^4}-G\left(k_m^2+\frac{n^2}{R^2}\right)+\rho_t\omega^2\right\}W=0 \tag{10c}$$

Making the arrangement of terms in the Eqs. (10a)-(10c), the shell frequency equation framed in the eigenvalue form as below

$$\left\{\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\rho_t\omega^2+\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}2\rho_t\omega+\begin{pmatrix}d_{11}&d_{12}&d_{13}\\d_{21}&d_{22}&d_{23}\\d_{31}&d_{32}&d_{33}\end{pmatrix}\right\}\begin{pmatrix}U\\V\\W\end{pmatrix}=0 \tag{11}$$

5. Results and discussions

In this section, the versatile numerical technique of wave propagation method has been used in current study to study the frequency analysis of functionally graded

Table 1 Frequency comparison of clamped-clamped cylindrical shell (ν=0.3, L/R=20, h/R=0.002)

m	n	Zhang *et al.* (2001)	Present
	1	0.03487	0.03487
	2	0.01176	0.01176
1	3	0.00708	0.00708
	4	0.00902	0.00902
	5	0.01377	0.01377
	1	0.08742	0.08742
	2	0.03155	0.03155
2	3	0.01586	0.01586
	4	0.01224	0.01224
	5	0.01482	0.01482

Table 2 Frequency comparison of natural frequencies (Hz) for a clamped-free cylindrical shell (m=1, L=502 mm, R=63.5 mm, h=1.63 mm, ν=0.28, $E=2.1\times10^{11}$ N m^{-2}, $\rho=7.8\times10^{3}$ Kg m^{-3})

n	Loy and Lam (1997)	Present
2	319.5	294.7
3	769.9	766.9
4	1465.8	1464.5
5	2367.1	2365.9
6	3470.3	3469.0

Table 3 Frequency of wave number versus for simply supported – simply supported cylindrical shell (m=1, L=2, h=0.001, R=1, N=0.5, $G=1\times10^{6}$ N−m)

n	Type I	Type II
1	13.57331	482.7318
2	4.420372	275.6446
3	2.356774	163.2589
4	2.432712	103.8762
5	3.511083	70.75681
14	2.829642	2.651830
15	3.251456	2.950367

cylindrical shells. For the convergence rate of cylindrical shell, the non-dimensional frequency enumerated in the current work. The proposed model based on wave propagation method with the combination Pasternak foundation is applied in order to accurately predict the acquired results of material data point. Table 1 shows the comparison of present results with Zhang *et al.* (2001) and in Table 2, the frequency results are well matched those evaluated by Loy and Lam (1997) for different modal numbers for C-F shells. Here, using present happened to be in a good consistency along with the so-called exact results furnished by Zhang *et al.* (2001) and Loy and Lam (1997).

Fig. 2 indicates the illustration of functionally graded cylindrical shell with Pasternak foundation. Table 3 indicates that the frequency values versus circumferential wave number. The frequencies are taken for circumferential modes n=1~15 and m=1. It is observed that the frequencies first increases and after decreases and pronounces again on enhancing wave number. It is due membrane and flexural stiffness of the shell. Tables 4-5 shows the variations of clamped - free frequencies versus the L/R and h/R for FG-CS. In Table 4, natural frequencies (Hz) with thickness to

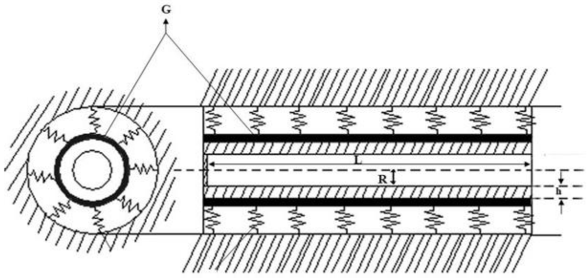

Fig. 2 Module of Pasternak foundation for FG-CSs

Table 4 Frequency with thickness-to-radius ratio for clamped-free (L=5, m=1, n=1, N=1, $G=1\times10^{6}$ N−m)

h/R	Type I	Type II
0.001	154.8270	154.8247
0.002	154.8516	154.8471
0.003	154.8763	154.8694
0.004	154.9009	154.8918
0.005	154.9255	154.9142

Table 5 Frequency with length-to-radius ratio for clamped-free (m=1, N=1, h=0.02, n=1, $G=1\times10^{6}$ N−m)

L/R	Type I	Type II
1	476.6915	476.6206
2	214.0532	213.9909
3	117.2077	117.1744
4	72.8583	72.8383
5	49.25787	49.2447

Table 6. Frequency with Pasternak foundation for clamped –simply supported cylindrical shell (L=2, h=0.001, m=1, n=1, R=1, N=1)

G (N-m)	Type-I	Type-II
1×10^{6}	560.9983	560.9978
2×10^{6}	613.6066	613.6059
3×10^{6}	657.1725	657.1717
4×10^{6}	693.2911	693.2902
5×10^{6}	723.1749	723.1738
6×10^{6}	747.8455	747.8442
7×10^{6}	768.1983	768.1968
8×10^{6}	785.0152	785.0137
9×10^{6}	798.9635	798.9618
10×10^{6}	810.5972	810.5955

Table 7 Frequency with Pasternak foundation for clamped – free cylindrical shell (L=2, h=0.001, m=1, n=1, R=1, N=1)

G (N-m)	Type-I	Type-II
1×10^{6}	543.8341	543.8336
2×10^{6}	599.4479	599.4473
3×10^{6}	645.4328	645.4320
4×10^{6}	683.5631	683.5622
5×10^{6}	715.1338	715.1327
6×10^{6}	741.2109	741.2096
7×10^{6}	762.7236	762.7222
8×10^{6}	780.4868	780.4853
9×10^{6}	795.2007	795.1990
10×10^{6}	807.4513	807.9496

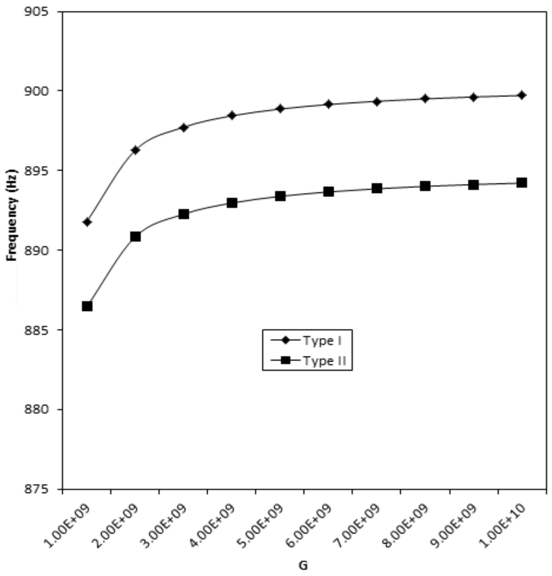

Fig. 3 Variation of natural frequencies (Hz) for C-C shell with G (K=0, L=2, h=0.001, m=1, n=1, R=1, N=1)

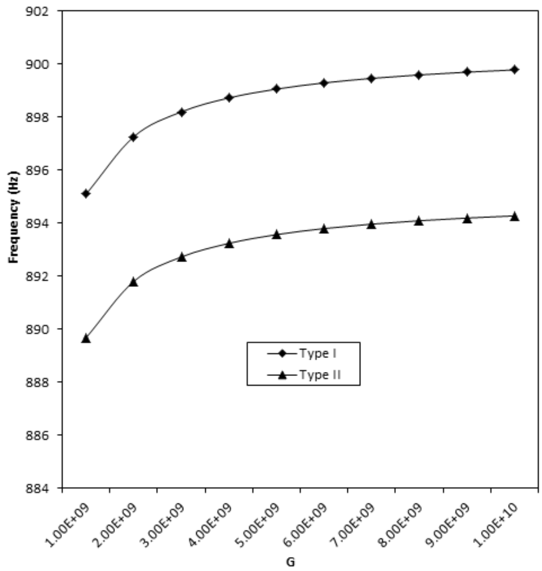

Fig. 4 Variation of natural frequencies (Hz) for SS-SS shell with G (L=2, h=0.001, m=1, n=1, R=1, N=1)

radius (h/R) for Pasternak elastic foundation G=1×10^6 (N-m) with Type-I and Type-II are tabulated. With increase in values of h/R, the frequency increases. It is noted that with Pasternak foundation, on increases h/R frequencies increases as for other cases. Table 5 shows that there is a smaller change in the frequencies when the value of L/R goes to higher step by step in both Types (I & II). The natural frequencies (Hz) versus length to radius ratio (L/R) for a cylindrical shell with Pasternak elastic foundation G=1×10^6 (N-m) is depicted in Table 5. It is noted that shell frequencies grow as L/R is enhanced i.e., as the shell becomes longer. It is observed that from Tables 4 and 5, the frequencies of Type-I is greater than Type-II with Pasternak foundation G. It is due to the material which is used for vibrating shell. Tables 6 and 7 depicts the frequencies with the variation of Pasternak foundation G=1×10^6~10×10^6 of FG-CSs with BCs C-S and C-F. The frequencies in Table 6 are tabulated with Type-I and Type-II. The boundary condition C-F is lower than that of C-S. These frequencies gain maximum value with the increase of Pasternak foundation G. The frequencies of Type-II is bit less from Type-I due to the constitute materials. It is noted that the frequency have great impact on placing the FG-CS on the Pasternak foundation.

Figs. 3 and 4 plots the graph of FG-CSs with two different BCs SS-SS and C-C. The frequencies in these figures are drawn with Type-I and Type-II versus Pasternak foundation values G. It can be seen that, the SS-SS frequencies are lower than that of C-C. For these two conditions, frequency variations show same behavior with Types I and -II. The frequencies are visible and increases for these two boundary conditions for first three Pasternak foundation values G=1×10^6~3×10^6. These frequencies first increases and gain maximum value with the increase of Pasternak foundation. For clamped-clamped conditions, variations of frequencies are higher than that of SS-SS conditions. For G=4×10^6~10×$10^6$4~10, a symmetrical behavior for natural frequencies is seen with proposed boundary conditions. It can be observed that Type-II frequencies are smaller than that of Type-I. It is due to the inducting of material in the shell vibration. The frequencies are affected on inhaling the foundation in the cylinder.

6. Conclusions

In present study, vibrations of functionally grade cylindrical shells have been investigated for the distribution of material composition of material with two categories with Pasternak elastic foundation. Here the wave propagation procedure has been applied to derive the shell frequency equation to represent this phenomenon. The frequencies are higher for higher values of circumferential wave number. The frequency behavior is investigated for circumferential wave number, height and length-to-radius ratios. Also the variations have been plotted against the different values of Pasternak foundation. The frequency pattern is found for the increasing and decreasing for height and length-to-radius ratios. The frequency first increases and gain maximum value with the increase for circumferential wave mode. It has been investigated that the frequencies get higher on implicating the elastic foundation of Pasternak. Stability of a cylindrical shell depends highly on these aspects of material. More the shell material sustains a load due to physical situations, the more the shell is stable. Any predicted fatigue due to burden of vibrations is evaded by estimating their dynamical aspects. For future concerns, the present model can be done for investigating the rotating FG-shells with Winkler model.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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