

Effects of elastic medium on buckling of microtubules due to bending and torsion

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Abstract. Microtubules buckle under bending and torsion and this property has been studied for free microtubules before using orthotropic elastic shell model. But as microtubules are embedded in other elastic filaments and it is experimentally showed that these elastic filaments affect the critical buckling moment and critical buckling torque of the microtubules. To prove that, we developed orthotropic Winkler like model and demonstrated that the critical buckling moment and critical buckling torque of the microtubules are orders of higher magnitude than those found for free microtubules. Our results show that Critical buckling moment is about 6.04 nNm for which the corresponding curvature is about $\theta = 1.33 \text{ rad}/\mu\text{m}$ for embedded MTs, and critical buckling torque is 0.9 nNm for the angle of $1.33 \text{ rad}/\mu\text{m}$. Our results well proved the experimental findings.

Keywords: microtubule; orthotropic material; buckling; bending; torsion; winkler like model; orthotropic elastic shell model; wave propagation approach

1. Introduction

Microtubules (MTs), first proposed the name to small filaments in 1961 (Bulinski and Borisy 1980) are long hollow cylindrical protein organelles with inner and outer diameters of about 20 to 30 nm respectively, present in almost all living cells, made up of subunits for α and β -tubulins molecules, capable of changing length by assembly or disassembly of their subunits (Howard 2001). MTs are sensitive to cold and several specific chemicals such as colchicine and other building proteins, complex assemblies like the mitotic spindle, centrioles, cilia and flagella, axonemes, neurotubules (Schliwa and Woehlke 2003).

They appeared to give strength and maintain the cell shape, mediate in cell motility and in the displacements of chromosomes at mitosis and meiosis and provide the path for vesicular transport (Chen and Chen 2000). They form the moving core of cilia and flagella. It is proved in many experiments that the deformation of MTs is the result of chemical reaction. For example, rise in tension on cells of nervous system leads to their assembly and increase in local curvature that result in their disintegration (Zheng *et al.* 1993, Odde and Renn 1999). Mechanical characteristics are crucial for complete understanding of biological processes

of MTs. Bending in MTs may occur due to several physiological processes in living cells like polymerization, acto-myosin contractility, motor activity especially in ciliary and flagellary motion (Waterman-Storer and Salmon 1997).

It was revealed in the experiments that during its biological functions, bending of protein MTs in fibroblast cell with a mean curvature of $0.4 \text{ rad}/\mu\text{m}$ is observed (Odde and Renn 1999). By applying some form of bending to MTs, other mechanical characteristics like flexural rigidity of MTs are also calculated (Felgner and Schliwa 1996). Salah *et al.* (2019) employed a simple four-variable integral plate theory for examining the thermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Hussain and Naeem (2017) examined the frequencies of armchair tubes using Flügge's shell model. The effect of length and thickness-to-radius ratios against fundamental natural frequency with different indices of armchair tube was investigated. Kolahchi and Cheraghbak (2017) studied with the nonlocal dynamic buckling analysis of embedded microplates reinforced by single-walled carbon nanotubes (SWCNTs). The material properties of structure are assumed viscoelastic based on Kelvin-Voigt model. Agglomeration effects are considered based on Mori-Tanaka approach. The elastic medium is simulated by orthotropic visco-Pasternak medium. Hussain *et al.* (2017) demonstrated an overview of Donnell theory for the

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frequency characteristics of two types of SWCNTs. Fundamental frequencies with different parameters have been investigated with wave propagation approach. Kolahchi (2017) investigated the bending, buckling and buckling of embedded nano-sandwich plates based on refined zigzag theory (RZT), sinusoidal shear deformation theory (SSDT), first order shear deformation theory (FSDT) and classical plate theory (CPT). In order to present a realistic model, the material properties of system are assumed viscoelastic using Kelvin–Voigt model. Hussain and Naeem (2018a) used Donnell’s shell model to calculate the dimensionless frequencies for two types of single-walled carbon nanotubes. The frequency influence was observed with different parameters. Bilouei *et al.* (2016) used as concrete the most usable material in construction industry it’s been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modelled with Euler-Bernoulli beam theory. Fatahi Vajari *et al.* (2019) studied the vibration of single-walled carbon nanotubes based on Galerkin’s and homotopy method. This work analyses the nonlinear coupled axial–torsional vibration of single-walled carbon nanotubes (SWCNTs) based on numerical methods. Two-second order partial differential equations that govern the nonlinear coupled axial–torsional vibration for such nanotube are derived. Kolahchi *et al.* (2016a) concerned with the dynamic stability response of an embedded piezoelectric nanoplate made of polyvinylidene fluoride (PVDF). In order to present a realistic model, the material properties of nanoplate are assumed viscoelastic using Kelvin–Voigt model. The visco-nanoplate is surrounded by viscoelastic medium which is simulated by orthotropic visco-Pasternak foundation. The PVDF visco-nanoplate is subjected to an applied voltage in the thickness direction. Asghar *et al.* (2019a, b) conducted the vibration of nonlocal effect for double-walled carbon nanotubes using wave propagation approach. Many material parameters are varied for the exact frequencies of many indices of double-walled carbon nanotubes. Demir *et al.* (2016) deals with buckling analysis of simply supported conical panels based on the Donnell’s shell theory. Different material properties have been considered such as isotropic, composite laminated and functionally graded (FG). The governing differential equation for buckling of laminated conical panel is derived. These equations are discrete using method of discrete singular convolution (DSC). Shannon’s delta kernel is used for trial functions. Arani and Kolahchi (2016) used a concrete material in construction industry it’s been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modelled with Euler-Bernoulli and Timoshenko beam theories. The characteristics of the equivalent composite being determined using mixture rule.

The foundation around the column is simulated with spring and shear layer. Sharma *et al.* (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Zamanian *et al.* (2017) considered the use of nanotechnology materials and applications in the construction industry. However, the nonlinear buckling of an embedded straight concrete columns reinforced with silicon dioxide (SiO_2) nanoparticles is investigated in the present study. The column is simulated mathematically with Euler-Bernoulli and Timoshenko beam models. Agglomeration effects and the characteristics of the equivalent composite are determined using Mori-Tanaka approach. The foundation around the column is simulated with spring and shear layer. Bensattalah *et al.* (2019) studied the critical buckling of a single-walled carbon nanotube (SWCNT) embedded in Kerr’s medium. Based on the nonlocal continuum theory and the Euler-Bernoulli beam model. The governing equilibrium equations are acquired and solved for CNTs subjected to mechanical loads and embedded in Kerr’s medium. Kerr-type model is employed to simulate the interaction of the (SWNT) with a surrounding elastic medium. Kolahchi *et al.* (2017) studied the dynamic buckling of sandwich nano plate (SNP) subjected to harmonic compressive load based on nonlocal elasticity theory. The material properties of each layer of SNP are supposed to be viscoelastic based on Kelvin-Voigt model. In order to mathematical modeling of SNP, a novel formulation, refined Zigzag theory (RZT) is developed. Furthermore, the surrounding elastic medium is simulated by visco-orthotropic Pasternak foundation model in which damping, normal and transverse shear loads are taken into account. Motezaker and Eyvazian (2020) deals with the buckling and optimization of a nanocomposite beam. The agglomeration of nanoparticles was assumed by Mori-Tanaka model. The harmony search optimization algorithm is adaptively improved using two adjusted processes based on dynamic parameters. The governing equations were derived by Timoshenko beam model by energy method. The optimum conditions of the nanocomposite beam- based proposed AIHS are compared with several existing harmony search algorithms. Kolahchi and Bidgoli (2016) presented a model for dynamic instability of embedded single-walled carbon nanotubes (SWCNTs). SWCNTs are modeled by the sinusoidal shear deformation beam theory (SSDBT). The modified couple stress theory (MCST) is considered in order to capture the size effects. The surrounding elastic medium is described by a visco-Pasternak foundation model, which accounts for normal, transverse shear, and damping loads. The motion equations are derived based on Hamilton’s principle. Madani *et al.* (2016) presented vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT)-reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions. The structure is subjected to an applied voltage in thickness direction which operates in control of vibration behavior of system. Kolahchi *et al.* (2016b) investigated the nonlinear

dynamic stability analysis of embedded temperature-dependent viscoelastic plates reinforced by single-walled carbon nanotubes (SWCNTs). The equivalent material properties of nanocomposite are estimated based on the rule of mixture. For the carbon-nanotube reinforced composite (CNTRC) visco-plate, both cases of uniform distribution (UD) and functionally graded (FG) distribution patterns of SWCNT reinforcements are considered. The surrounding elastic medium is modeled by orthotropic temperature-dependent elastomeric medium. The viscoelastic properties of plate are assumed based on Kelvin–Voigt theory. Batou *et al.* (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. Motezaker and Kolahchi (2017a) Investigated the Seismic response of the concrete column covered by nanofiber reinforced polymer (NFRP) layer. The concrete column is studied in this paper. The column is modeled using sinusoidal shear deformation beam theory (SSDT). Mori-Tanaka model is used for obtaining the effective material properties of the NFRP layer considering agglomeration effects. Using the nonlinear strain-displacement relations, stress-strain relations and Hamilton's principle, the motion equations are derived. Motezaker and Kolahchi (2017b) presented the dynamic analysis of a concrete pipes armed with Silica (SiO_2) nanoparticles subjected to earthquake load. The structure is modeled with first order shear deformation theory (FSDT) of cylindrical shells. Mori-Tanaka approach is applied for obtaining the equivalent material properties of the structure considering agglomeration effects. Akgöz and Civalek (2014) presented a new microstructure-dependent sinusoidal beam model for buckling of microbeams using modified strain gradient theory. This microbeam model can take into consideration microstructural and shear deformation effects. The equilibrium equations and corresponding boundary conditions in buckling are derived with the minimum total potential energy principle. Kolahchi *et al.* (2017) focussed with general wave propagation in a piezoelectric sandwich plate. The core is consisted of several viscoelastic nanocomposite layers subjected to magnetic field and is integrated with viscoelastic piezoelectric layers subjected to electric field. The piezoelectric layers play the role of actuator and sensor at the top and bottom of the core, respectively. Benmansour *et al.* (2019) analyzed the dynamic and bending behaviors of isolated protein microtubules. Microtubules (MTs) can be considered as bio-composite structures that are elements of the cytoskeleton in eukaryotic cells and posses considerable roles in cellular activities. They have higher mechanical characteristics such as superior flexibility and stiffness. On the other hand, application of bending deformation caused by hydrodynamic flow were applied to calculate the flexural rigidity of MTs by studying the relaxation process of MTs bending through laser trap or deduced from thermal fluctuation of MTs shapes (Felgner *et al.* 1997, Mickey and

Howard 1995). Motezaker *et al.* (2020) presented the present research post-buckling of a cut out plate reinforced through carbon nanotubes (CNTs) resting on an elastic foundation. Material characteristics of CNTs are hypothesized to be altered within thickness orientation which is calculated according to Mori-Tanaka model. For modeling the system mathematically, first order shear deformation theory (FSDT) is applied and using energy procedure, the governing equations can be derived. Khelifa *et al.* (2018) deals with buckling analysis with stretching effect of functionally graded carbon nanotube-reinforced composite beams resting on an elastic foundation. The single-walled carbon nanotubes (SWCNTs) are aligned and distributed in polymeric matrix with different patterns of reinforcement. The material properties of the CNTRC beams are estimated by using the rule of mixture. For the accurate measurement of bending rigidity, it is necessary that during bending, buckling must not occur so the critical buckling load should be known for MTs. In this regard, some works have been done about bending buckling of MTs. Many simulations on bending buckling were developed with the help of elastic sheet model and three-dimensional finite model which were related to the effect of MTs helical structure (Janosi *et al.* 2000, Varga *et al.* 2007). It was also demonstrated that orthotropic walls have great effect on buckling of cylindrical shells. The effect of wall orthotropy on critical buckling parameters was discussed by Li (2008) using orthotropic shell model for free MTs. As MTs, observed by immunofluorescence, extend sometimes further than the cell boundaries, and are straighter than usual, suggesting that the curvature of MT in living cells results from the interaction of MT with MAPs or other structures like intermediate filaments (Woody *et al.* 1982), (Bayley *et al.* 1982). Also in (Grishchuk *et al.* 2005) it was demonstrated that critical buckling load may increase by hundreds in living cells as compared to buckling force for free MTs. Therefore it is the need to develop a theoretical model to study the bending buckling behavior of the MTs. Motezaker *et al.* (2020) analysis the vibration, buckling and bending of annular nanoplate integrated with piezoelectric layers at the top and bottom surfaces. The higher order nonlocal theory for size effect and Gurtin–Murdoch theory for surface effects are utilized. The governing equations are derived based on the layer-wise (LW) theory and Hamilton's principle. The differential method (DCM) as a new numerical procedure, is utilized to solve the motion equations for obtaining the frequency, buckling load and deflection.

Several researchers used different approaches for the investigation of frequency of cylinders, plates, beams and concrete material (Kagimoto *et al.* 2015, Mesbah and Benzaid 2017, Alijani and Bidgoli 2018, Demir and Livaoglu 2019, Samadvand and Dehestani 2020, Ayat *et al.* 2018, Behra *et al.* 2018, Narwariya *et al.* 2018, Rezaiee-Pajand *et al.* 2018, Sedighi and Sheikhanzadeh 2017). Recently Hussain and Naeem (2019a, b, c, d, 2020a) performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. They investigated many physical parameters for the rotating and non-rotating vibrations of armchair, zigzag and chiral

indices. Moreover, the mass density effect of single walled carbon nanotubes with in-plane rigidity has been calculated for zigzag and chiral indices. Many material researchers calculated the frequency of nano structure using different techniques, for example, Timoshenko beam model (Zidour *et al.* 2014), SiO₂ nanoparticles (Zarei *et al.* 2017, Amnieh *et al.* 2018, Jassas *et al.* 2019), layerwise theory (Hajmohammad *et al.* 2018a, Hajmohammad *et al.* 2019), Flugge shell theory (Zidour *et al.* 2014), Grey Wolf algorithm (Kolahchi *et al.* 2020), reinforced polymer layer (Hajmohammad *et al.* 2018b), agglomerated CNTs (agglomerated CNTs), zigzag theory (Kolahchi *et al.* 2017), and viscoelastic cylindrical shell (Hosseini and Kolahchi 2018, Hajmohammad *et al.* 2018c).

In this article, we developed an orthotropic –Winkler like model to investigate the buckling characteristics of embedded MTs upon bending and torsion. We compared the critical buckling parameters for free MTs (Yi *et al.* 2008) and embedded MTs and found that the critical buckling moment is about 6.04 nN nm for which the corresponding curvature is about $\theta = 1.33 \text{ rad}/\mu\text{m}$ for embedded MTs, and critical buckling torque is 0.9 nN nm for the angle of $1.33 \text{ rad}/\mu\text{m}$.

2. Materials and methods

Over the past several years, vibration of nanostructures of various configurations and boundary conditions have been extensively studied (Hussain *et al.* 2018a, Hussain *et al.* 2018b, Hussain *et al.* 2018c, Hussain and Naeem 2018b, Hussain *et al.* 2019a, Hussain *et al.* 2019b, Hussain *et al.* 2020a, Hussain and Naeem 2020b, Asghar *et al.* 2020, Hussain *et al.* 2020b, c, d, e, f, g, Taj *et al.* 2020a, Taj *et al.* 2020a, b, c). We will apply Orthotropic Elastic Shell Model to analyze the buckling of MTs under bending and torsion. Surrounding medium of MTs will be modeled by Winkler model. We will develop orthotropic Winkler-like model by the combination of these models. We will use wave propagation approach to solve the developed model.

2.1 Orthotropic elastic shell model for MTs

In this section, we developed an “Orthotropic elastic shell model” for buckling of MTs within an elastic medium due to bending and torsion. Since this model has four independent material constants, longitudinal modulus, circumferential modulus, shear modulus, and Poisson ratio along the longitudinal direction (Ventsel and Krauthammer 2004), denoted by E_x , E_θ , $G_{x\theta}$, and ν_x respectively (Sirenko *et al.* 1996). The cross section of MTs will be treated as an equivalent circular annular shape with equivalent thickness about $h \approx 2.7 \text{ nm}$ (de Pablo *et al.* 2003). Thus the elastic moduli, in-plan stiffnesses and the mass density are found based on such a thickness, $h \approx 2.7 \text{ nm}$. The bending thickness of MTs can be calculated by using so called the “bridge” thickness, 1.1 nm (de Pablo *et al.* 2003), which is much smaller than equivalent thickness $h \approx 2.7 \text{ nm}$. Thus, just like the single walled carbon nanotubes (Flügge 2013) the effective bending stiffness of MTs, modeled as elastic shell, should be considered to be

Table 1 Numerical values of material parameters for MTs

Parameters	Symbols	Values
Longitudinal modulus	E_x	$0.5 - 2.0 \times 10^9 \text{ pa}$
Circumferential modulus	E_θ	$1.0 - 4.0 \times 10^6 \text{ pa}$
Shear modulus in $x\theta$ plane	$G_{x\theta}$	$1.0 \times 10^6 \text{ pa}$
Longitudinal Poisson ratio	ν_x	0.3
Equivalent thickness	h	$2.7 \times 10^{-9} \text{ m}$
Mass density per unit volume	ρ	1.47 g/cm^3

an independent material constant. The bending stiffness of MTs can be estimated by effective thickness which is about 1.6 nm (de Pablo *et al.* 2003). The range of the values of these material constants for MTs is identified from the data available in literatures (Kawaguchi *et al.* 2008) and summarized in Table 1.

2.2 Winkler like model

Upon incipient bending buckling and torsional buckling of MTs, the surrounding filamentous network of cytoskeleton is deformed. In turn, the surrounding fibres exert a distributed force on MTs in the opposite direction of the bending buckling and torsional buckling. Inspired by the valid application of Winkler-like model to the buckling of MTs due to axial and radial force (Taj and Zhang 2011) and on the buckling of Carbon Nanotubes (Ru 2000), we used this model to relate the effects of surrounding on bending buckling and torsional buckling of MTs.

The said model reads as

$$P = -Kw \quad (1)$$

Where negative sign shows that the pressure ‘P’ is opposite to the incipient buckling mode and ‘K’ is the Elastic constant of fibres surrounding the MTs.

2.3 Orthotropic Winkler like model

Combining the above mentioned orthotropic and Winkler models, we developed orthotropic Winkler-like model for the buckling of MTs due to forces N_x , N_θ and $N_{x\theta}$, as Orthotropic elastic shell model is described by following three equations (Eslami and Javaheri 1999).

$$\begin{aligned} F_1 &= A_1 u + B_1 v + C_1 w = 0, \\ F_2 &= A_2 u + B_2 v + C_2 w = 0, \\ F_3 &= A_3 u + B_3 v + C_3 w = 0. \end{aligned} \quad (2)$$

Where

$$\begin{aligned} A_1 &= (K_x + N_x)R^2 \frac{\partial^2}{\partial x^2} + 2RN_{x\theta} \frac{\partial^2}{\partial x \partial \theta} \\ &\quad + \left(\frac{K_{x\theta}R^2 + D_{x\theta}}{R^2} + N_\theta \right) \frac{\partial^2}{\partial \theta^2}, \\ B_1 &= R(\nu_x K_\theta + K_{x\theta}) \frac{\partial^2}{\partial x \partial \theta}, \\ C_1 &= -R(\nu_\theta K_x - N_\theta) \frac{\partial}{\partial x} + RD_x \frac{\partial^3}{\partial x^3} - \frac{D_{x\theta}}{R} \frac{\partial^3}{\partial x \partial \theta^2}, \\ A_2 &= R(\nu_\theta K_x + K_{x\theta}) \frac{\partial^2}{\partial x \partial \theta}, \\ B_2 &= (K_\theta + N_\theta) \frac{\partial^2}{\partial \theta^2} + 2RN_{x\theta} \frac{\partial^2}{\partial x \partial \theta} \end{aligned}$$

$$\begin{aligned}
& + \left(\frac{K_{x\theta} R^2 + 3D_{x\theta}}{R^2} + N_x \right) R^2 \frac{\partial^2}{\partial x^2}, \\
C_2 = & -(K_\theta + N_\theta) \frac{\partial}{\partial \theta} - 2RN_{x\theta} \frac{\partial}{\partial x} + (v_\theta D_x + 3D_{x\theta}) \frac{\partial^3}{\partial x^2 \partial \theta}, \\
A_3 = & R(v_\theta K_x - N_\theta) \frac{\partial}{\partial x} - RD_x \frac{\partial^3}{\partial x^3} + \frac{D_{x\theta}}{R} \frac{\partial^3}{\partial x \partial \theta^2}, \\
B_3 = & (K_\theta + N_\theta) \frac{\partial}{\partial \theta} + 2RN_{x\theta} \frac{\partial}{\partial x} - (v_\theta D_x + 3D_{x\theta}) \frac{\partial^3}{\partial x^2 \partial \theta}, \\
& (2v_\theta D_x + 4D_{x\theta}) \frac{\partial^4}{\partial x^2 \partial \theta^2} - \frac{D_\theta}{R^2} \left(\frac{\partial^2}{\partial \theta^2} + 1 \right)^2 \\
C_3 = & -R^2 D_x \frac{\partial^4}{\partial x^4} + 2RN_{x\theta} \frac{\partial^2}{\partial x \partial \theta} - \\
& + N_\theta \frac{\partial^2}{\partial \theta^2} + N_x R^2 \frac{\partial^2}{\partial x^2} - K_\theta.
\end{aligned}$$

Here longitudinal coordinate is represented by x and circumferential coordinate is θ , and u, v and w are axial, circumferential and inward displacements respectively, density is represented by ρ and R is the average radius. Furthermore, longitudinal and circumferential Poisson ratios and Young's moduli are denoted by ν_x, ν_θ and E_x, E_θ respectively which satisfy the relation $\nu_\theta/\nu_x = E_\theta/E_x$, while shear modulus is denoted by $G_{x\theta}$, in plane stiffness in longitudinal direction is denoted by $K_x [= E_x h/(1 - \nu_x \nu_\theta)]$ and in circumferential direction is $K_\theta [= E_\theta h/(1 - \nu_x \nu_\theta)]$, and $K_{x\theta} (= G_{x\theta} h)$ is in plane shear. Furthermore effective bending stiffness is represented by D_x, D_θ , and $D_{x\theta}$ respectively (Flügge 2013).

Bending stiffness of MTs is measured by "bridge" thickness because of the latticed structure of MTs which is smaller than bending stiffness. We considered effective bending stiffness of MTs as independent material constant during modeling as an elastic shell, in contrary to elastic shell theory based on the thickness of MTs. substituting

$$\alpha = \nu_\theta/\nu_x = E_\theta/E_x = K_\theta/K_x = D_\theta/D_x,$$

and

$$\beta = G_{x\theta}/E_x \approx G_{x\theta}/E_x (1 - \alpha \nu_x^2) = D_{x\theta}/D_x = K_{x\theta}/K_x, \quad (\alpha \nu_x^2 \rightarrow 0),$$

The orthotropic elastic shell model can be described by four parameters E_x, ν_x, α , and β . It can be easily verified that isotropic elastic shell model can be derived from orthotropic elastic shell by using $\alpha = 1$ and $\beta = 1(1 - \nu)/2$.

2.4 Buckling of MTs upon bending

The buckling mode upon bending can be taken as (Yi *et al.* 2008)

$$\begin{aligned}
u(x, \theta) &= \cos\left(\frac{m\pi}{L}x\right) \sum_{n=1}^{\infty} A_n \cos(n, \theta), \\
v(x, \theta) &= \sin\left(\frac{m\pi}{L}x\right) \sum_{n=1}^{\infty} B_n \sin(n, \theta), \\
w(x, \theta) &= \sin\left(\frac{m\pi}{L}x\right) \sum_{n=1}^{\infty} C_n \cos(n, \theta). \quad (3)
\end{aligned}$$

In which A_n, B_n and C_n are real constants, n represent the circumferential wave number, $m (\neq 0)$ half axial wave number, L is the length of MT, and the dimensionless axial wavelength calculated as $L/(Rm)$.

By the combination of (1), (2) and (3), we obtained the "Winkler like Model" for MTs with in an elastic medium.

Now writing the system of equations, in matrix form we obtained

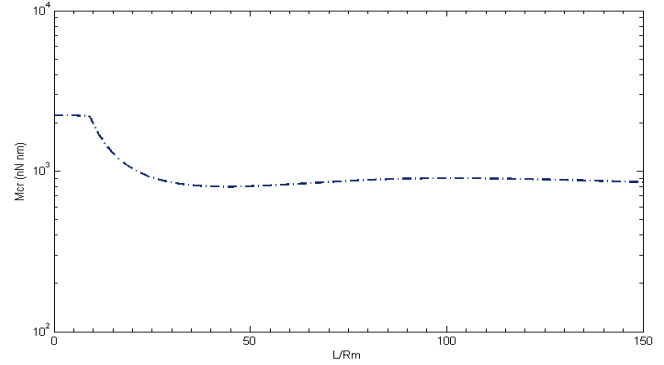


Fig. 1 Dependence of critical buckling M_{cr} on the dimensionless axial wavelength (normalized by diameter $2R$) L/Rm is obtained for isotropic shell model ($\alpha=1$, $\beta=0.35$)

$$\mathbf{M} \left(M_{cr}, K, \frac{L}{Rm} \right)_{9 \times 9} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ B_1 \\ B_2 \\ B_3 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = 0 \quad (4)$$

We are looking for nontrivial solution which leads to $\det \mathbf{M}=0$, which result to critical buckling load, hence the buckling mode.

3. Results

3.1 Buckling upon bending

Fig. 1 graphically demonstrate critical buckling moment M_{cr} plotted against normalized length L/Rm , where R is the radius of MTs and m is the half axial wave number. K shows the effects of surroundings on critical buckling of MTs in natural environment where they lie. Here it is clear that without considering the effects of surroundings, the critical buckling moment of MTs was about 0.85 nNm. In our work, where we considered the embedded MTs, this value increases upto 6.04 nNm which is about 7 times more than the value for free MTs. The proximity of elastic medium increases stiffness of MTs by a considerable amount which requires special attention that how the MTs when embedded in elastic medium provide the shape and rigidity to the cell.

This value of moment corresponds to the critical buckling curvature of about 0.16 rad/ μm , calculated by the expression $1/\rho = M_{cr}/\pi E_x h R^3$. We use the same value for longitudinal modulus as used for free MTs. Experimental value for critical buckling curvature was 0.4 rad/ μm (Odde *et al.* 1999), which is very close to our theoretical value. This proves that due to elastic effects on MTs, its rigidity increases and it can provide shape and rigidity to the cell to maintain its shape for proper

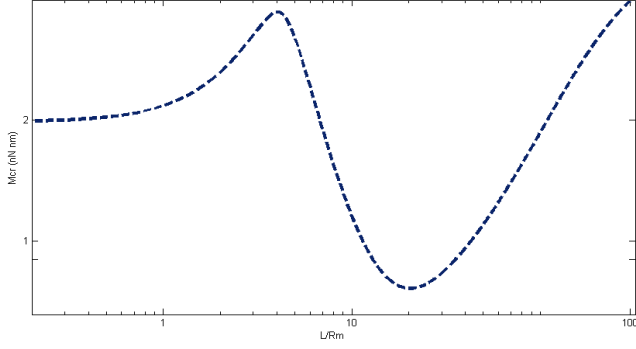


Fig. 2 The dependence of critical buckling M_{cr} on the dimensionless wavelength (normalized by the diameter $2R$). L/Rm is obtained for orthotropic model with $\alpha=\beta=0.001$

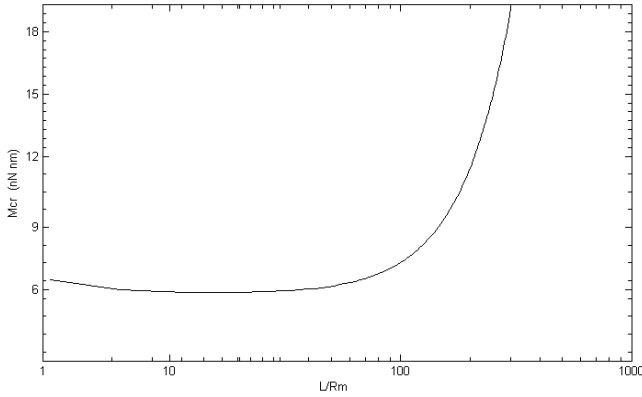


Fig. 3 Dependence of critical buckling M_{cr} on the dimensionless axial wavelength (normalized by diameter $2R$) L/Rm is obtained for orthotropic Winkler-like model

functioning of organelles of the cell.

3.2 Effects of elastic medium on torsional buckling of MTs

During many physiological processes such as in moving motor proteins along MTs, movement of cilia and flagella, movement of chromosomes, and crawling with skewed angle on the inner surface of plasma membrane, MTs rotate within the cell. Before torsion of MTs was studied without considering the effect of elastic medium (Yi *et al.* 2008). In this study, the authors demonstrate the critical buckling load due to torsion. But the surrounding medium may affect the torsional behaviour of MTs. Due to coupling with the surrounding; the critical buckling load due to torsion may rise. To confirm the above questions, we discussed in this paper the effect of medium on torsional mechanics of MTs. We calculated the buckling torque and corresponding critical torsional angle.

For a MT embedded in elastic medium, shearing force $N_{x\theta}$ is very vital. But $N_x = N_\theta = 0$, then the buckling mode for embedded MTs due to torsion can be represented by following (Flügge 2013).

$$u(x, \theta) = U \cos\left(\frac{m\pi}{L}x - n\theta\right)$$

$$v(x, \theta) = V \cos\left(\frac{m\pi}{L}x - n\theta\right)$$

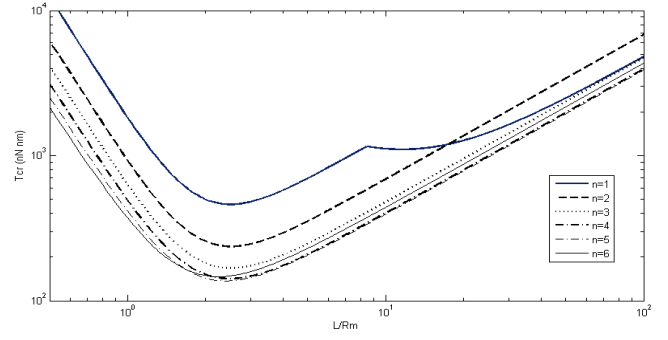


Fig. 4 Dependence of critical buckling torque T_{cr} on the dimensionless axial wavelength (normalized by diameter $2R$) L/Rm is obtained for isotropic elastic shell model ($\alpha=1$, $\beta=0.35$)

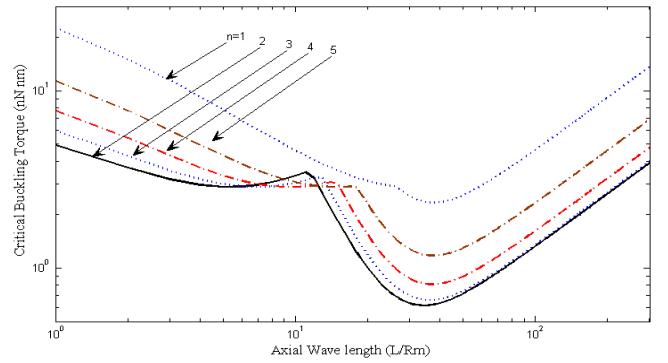


Fig. 5 Dependence of critical buckling torque T_{cr} on the dimensionless axial wavelength (normalized by diameter $2R$) L/Rm is obtained for anisotropic elastic shell model ($\alpha=\beta=0.001$)

$$w(x, \theta) = W \sin\left(\frac{m\pi}{L}x - n\theta\right) \quad (5)$$

In which U, V and W are real constants. n denotes the circumferential wave number, L is length of MT and nonzero ' m ' is half-axial wave number. Putting (1) and (5) into the orthotropic elastic shell model (2) and obtained the set of equations

$$\left[\frac{2\pi mn RN_{x\theta}}{L} - \frac{\pi^2 m^2 R^2 (K_x + N_x)}{L^2} - \frac{n^2 \{R^2 (K_{x\theta} + N_\theta) + D_{x\theta}\}}{R^2} \right] U +$$

$$\left[\frac{\pi mn R (K_{x\theta} + K_\theta v_\theta)}{L} \right] V$$

$$+ \left[\frac{\pi m \{n^2 L^2 D_{x\theta} + R^2 [L^2 (N_\theta - K_x v_\theta) - n^2 L^2 D_{x\theta}]\}}{L^3 R} \right] W = 0,$$

$$\left[\frac{2\pi mn RN_{x\theta}}{L} - \frac{n^2 \{R^2 (K_{x\theta} + N_\theta) + D_{x\theta}\}}{R^2} - n^2 (K_\theta + N_\theta) \right] V +$$

$$\left[\frac{\pi^2 m^2 n (3D_{x\theta} + D_{x\theta} v_\theta)}{L^2} - \frac{2\pi mn RN_{x\theta}}{L} + n (K_\theta + N_\theta) \right] W = 0,$$

$$\left[\frac{\pi m \{n^2 L^2 D_{x\theta} + R^2 [L^2 (N_\theta - K_x v_\theta) - \pi^2 m^2 D_{x\theta}]\}}{L^3 R} \right] U +$$

$$\left[\frac{\pi^2 m^2 n (3D_{x\theta} + D_{x\theta} v_\theta)}{L^2} - \frac{2\pi mn RN_{x\theta}}{L} - n (K_\theta + N_\theta) \right] V$$

$$- \left[\frac{(n^2 - 1)^2 D_\theta}{R^2} + \frac{R^2 [\pi^2 m^2 L^2 (4n^2 D_{x\theta} + R^2 N_x + 2n^2 D_{x\theta} v_\theta) + L^4 (2\pi RK + n^2 N_\theta) + \pi^4 m^4 R^2 D_x - 2\pi mn L^3 RN_{x\theta} + L^4 K_\theta]}{L^4 R^2} \right] W = 0 \quad (6)$$

We are interested in the nonzero solution of (6), which

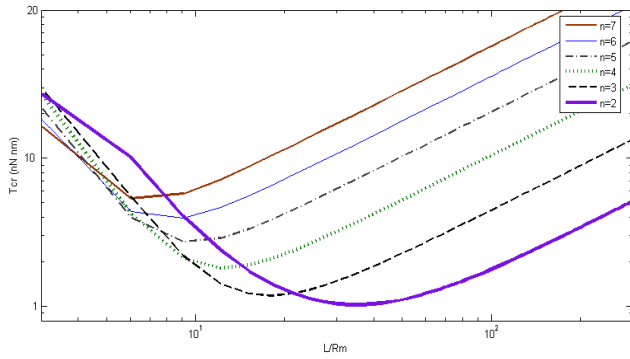


Fig. 6 Dependence of critical buckling torque T_{cr} on the dimensionless axial wavelength (normalized by diameter 2R) L/Rm is obtained for orthotropic Winkler-like model

lead to $\det \mathbf{M} = 0$ where

$$\mathbf{M} \left(T_{cr}, \frac{L}{Rm}, n \right)_{3 \times 3} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = 0 \quad (7)$$

3.3 Buckling due to torsion

Eq. (7) is the matrix form of (6). Eq. (6) are derived by setting $R = 13 \text{ nm}$, $E_x = 1 \text{ GPa}$, $\nu_x = 0.3$, $\alpha = 0.001$ and $\beta = 0.001$, the critical buckling torque T_{cr} with different length $L/(Rm)$ and n is plotted in Fig. 6. With the comparison of orthotropic elastic shell model for free MTs, it is clear that critical buckling torque is near 0.95 nNm, due to which the critical torsional angle is about $\theta = 1.33 \text{ rad}/\mu\text{m} \approx 76.24^\circ/\mu\text{m}$ and corresponds to a skew angle of filament about $\gamma = R\theta \approx 0.99^\circ$.

For a MT of significant length, $m=3$ and $n=2$ correspond to minimum buckling load. In this case, it can be verified from Eq. (7) that the force for critical buckling derived by relation $(N_{x\theta})_{cr} = \pi R E_x h / L (1 - \nu_x \nu_\theta)$. Moreover, the critical torque can be related as $T_{cr} = 2\pi R^2 (N_{x\theta})_{cr}$, with the help of this equation, critical buckling torque can be derived as $T_{cr} = 2\pi R^3 E_x h / L (1 - \nu_x \nu_\theta) \approx 2\pi^2 R^3 E_x h / L$. Our result pointed out that the embedded MTs are stiffer than the free MTs which were calculated earlier (Yi *et al.* 2008). But elastic medium in the surrounding of MTs significantly increase the rigidity of MTs which is not easy to ignore. Our results shows that embedded MTs can bear 12 times more force than free MTs.

4. Conclusions

We combined orthotropic elastic shell model with Winkler like model to develop orthotropic Winkler-like model to investigate the effects of elastic surrounding on the buckling behavior of MTs under bending and torsion. Critical buckling moment of about 6.04 nN nm is obtained to which the corresponding curvature is about $\theta = 1.33 \text{ rad}/\mu\text{m} \approx 76.24^\circ/\mu\text{m}$ for embedded MTs, critical buckling torque of 0.9 nN nm for the angle of $1.33 \text{ rad}/\mu\text{m}$ for a MT is derived. Our designed results of orthotropic Winkler-like model are compared with orthotropic elastic

shell model for free MTs. It is clear that, by orthotropic elastic shell model, we cannot obtain the values of the critical bending buckling, which are obtained by experiment (Odde *et al.* 1999). But our proposed model well agrees with the practical values obtained in the laboratory. Our calculation shows surrounding medium has drastic effect on the stiffness of MTs hence on the cells. In this paper, we tried to prove some experimental results about embedded MTs. We demonstrated theoretically that coupling of medium in which MTs lie and perform their function greatly affect the mechanical properties of MTs. Particularly, we used Orthotropic Winkler-like model for bending buckling and torsional buckling and proved that due to coupling, how bending moment and torsional buckling load increase and give strength to the cell.

In future, we can consider the non-linear and viscous effects of surrounding medium on MTs. Similar procedure can be applied to calculate the effect of medium on other components of cytoskeleton. We can also develop a mathematical model which can formulate all the components of cytoskeleton as all components together give shape and maintain the cell rigidity. The present study can be appropriate to employ for analyzing the Winkler's model with embedded microtubules using finite element method.

Declaration of conflicting interests

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