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Degradation reliability modeling of plain concrete for pavement under flexural fatigue loading

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Abstract. This study aims to establish a new methodological framework for the evaluation of the evolution of the reliability of plain concrete for pavement vs number of cycles under flexural fatigue loading. According to the framework, a new method calculating the reliability was proposed through probability simulation in order to describe a random accumulation of fatigue damage, which combines reliability theory, one-to-one probability density functions transformation technique, cumulative fatigue damage theory and Weibull distribution theory. Then the statistical analysis of flexural fatigue performance of cement concrete tested was carried out utilizing Weibull distribution. Ultimately, the reliability for the tested cement concrete was obtained by the proposed method. Results indicate that the stochastic evolution behavior of concrete materials under fatigue loading can be captured by the established framework. The flexural fatigue life data of concrete at different stress levels is well described utilizing the two-parameter Weibull distribution. The evolution of reliability for concrete materials tested in this study develops by three stages and may corresponds to develop stages of cracking. The proposed method may also be available for the analysis of degradation behaviors under non-fatigue conditions.

Keywords: pavement; cement concrete; fatigue damage; degradation reliability; Monte Carlo; Weibull distribution

1. Introduction

A large number of cement concrete pavements recently often fail to reach the service life, occurring serious structural distresses, meaning that a poor reliability of serving cement concrete pavement exists. After the deterioration of cement concrete pavement occurs, especially structural distresses, the condition of road will deteriorate rapidly, and then the maintenance and reconstruction will cost an arm and a leg (Chen and Tan 2011, Tang *et al*. 1996). Therefore, the construction of reliable and durable pavement structures to withstand the environment factor and vehicle load will be a long-term challenge. In general, the fixing of cement concrete pavement is more difficult, cumbersome and time-consuming compared to asphalt pavement (Xue *et al*. 2014). Service status of concrete pavement must be therefore evaluated to repair it in a more scientific and objective way, which is also of great guiding significance for highway administrations.

Usually, the presence of uncertain factors, such as its structural parameters, environment surrounded by and traffic loading subjected to pavement, results in the randomness of service condition and service life for pavement structures (Nejad *et al*. 2013). However, the existing evaluation methods of cement concrete pavements are mainly deterministic, and rarely consider related problems from an uncertain point of view, resulting in a poor evaluation result and a waste of maintenance costs. Taking into account that the reliability theory, a decision-making method to handle the issues on uncertainties in engineering domains, has a relatively perfect theory system, the reliability assessment of cement concrete pavement is necessarily conducted. The traditional probabilistic or reliability theory has been introduced into the pavement design and evaluation in the past (Darter 1976, Gao *et al*. 2009, Huang *et al*. 1993, Li *et al*. 1991, Liu *et al*. 2005, Luo *et al*. 2013, Luo *et al*. 2018, Tan *et al*. 2002). For example, Darter (1976) explained the important and necessary of application of probabilistic methods to the design and rehabilitation of pavement systems, and then outlined the methodology for practical and useful application. Li *et al*. (1991) presented the probabilistic models of cement concrete pavement design parameters and recommended reliability levels. Liu *et al*. (2005) established the cement concrete pavement design formula based on the fuzzy-random reliability. Gao *et al*. (2009) proposed the reinforcement ratio design method of continuously reinforced concrete pavement based on reliability. Nevertheless, some new problems or environments faced by pavement hinder the development of the traditional reliability theory, such as the emergence of new technologies/materials, the complication of pavement structures and its design, the diversification of evolution of status, and systematization of material/structure parameters. Hence, the reliability theory will encounter many difficulties in the practice of pavement engineering.

In general, most of failures of pavement presents a gradual failure process, while the traditional reliability

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theory revolves around sudden failure, which is not beneficial for the understanding of pavement failure law. Previous researches on pavement reliability (Huang *et al*. 1993, Luo *et al*. 2014, Tan *et al*. 2002) revolve around design reliability based on sudden failure. However, some researches indicated that service and mechanical performance of pavement degrade with service time (Bu *et al*. 2018, Chen and Won 2007, Gao *et al*. 2017, Gayana and Chandar 2018, Sadeghi and Hesami 2018, Tang *et al*. 2020). Therefore, it would be incomplete to only conduct the reliability evaluation and analysis in the design period without consideration of the decay process of performance.

Subjected to traffic loads and environmental effects, concrete pavements at work will be deteriorated in terms of cracks (Chen and Won 2007). One could make a reasonable explanation that external factors contribute to the propagation of initial cracks, ultimately leading to fatigue cracking of concrete pavement in macroscale. Earlier studies showed that the fatigue damage of concrete is usually a consequence of the initiation and propagation of crack within it (Akcay *et al*. 2016, Chen and Won 2007, Yang *et al*. 2019), and thus the cumulative fatigue damage (CFD) can indirectly reflect the developing process of cracks. In addition, the performance degradation of concrete in terms of the accumulation or evolution of fatigue damage is one of the main factors for a degradation in the reliability and durability of concrete slabs, which always results in other distresses. Furthermore, progressive damage data until a failure occurs in the concrete slabs may play an important role in concrete pavement maintenance decision-making. For most of concrete pavements, however, the data information during accumulation of fatigue damage is ignored in many existed design and evaluation methodologies. For these reasons, it is essential to explore the stochastic evolution behavior of CFD for cement concretes in pavement.

A number of CFD models were reported in the past literatures (Alliche and Frangois 1992, Fatemi and Yang 1998, Meng and Song 2007, Zhao and Sun 1999). These models are, however, deterministic without taking into account the randomness resulted from uncertainties. Considering that the CFD of engineering materials is random in nature, these models fail to actually characterized the degradation behaviors (i.e., CFD) of these materials. Generally, the randomness is the result of the inherent variability of internal micro-structures of materials and the uncertainty of repeated loads. In terms of probability and determinacy, a traditional deterministic method is unable to deal with the issues about variability or uncertainty and estimate the uncertain accumulation of fatigue damage accurately. Thus, a new general methodological framework is urgently need to be established in order to extrapolate the degradation behavior of materials utilizing probabilistic method. Meanwhile, the framework introduces a new perspective on the reliability evaluation for concrete pavements, breaking the technological bottleneck influencing the development of concrete pavement reliability. From the computing complexity point of view, a simple calculating method should be proposed in order to describe the randomness of CFD of engineering materials, which should be able to accomplish the reduction of the computing complexity. Furthermore, it allows to effectively calculate the reliability of engineering materials under flexural fatigue loading.

The purposes of this study are to establish a new methodological framework analyzing reliability; propose a new calculating method of reliability through a probabilistic simulation technique; statistically analyze the flexural fatigue performance of cement concrete in pavement; and ultimately assess the evolution of reliability for concrete under fatigue loading utilizing the proposed approach.

2. Methodological framework

Properties of pavement and its component materials are influenced by various of uncertainties (Luo *et al*. 2018, Nejad *et al*. 2013, Pei *et al*. 2016, Stubstad *et al*. 2002, Zhang *et al*. 2018). Similarly, the accumulation of fatigue damage for the tested materials also is affected by several uncertainties, such as the dispersion of internal microstructures in materials and applied loads, with the result that the accumulation of fatigue damage is a stochastic process in this study, which fails to be characterized in terms of a traditional deterministic method. To do this, it is imperative to analyze the accumulating process of fatigue damage through a reliability theory handling the issues on uncertainties.

To our knowledge, the data information in degradation process is not considered in the traditional reliability, the degradation reliability should be analyzed in order to more scientifically evaluate the safety of cement concrete slabs in pavement, making sure the sufficient service life of concrete pavement. Therefore, a methodological framework analyzing the reliability is established in this study to take full advantage of information in the degradation process (see Fig. 1). It is found that the methodological framework is divided into two parts: degradation system and reliability system. The primary theories involved in the framework include reliability theory, probability distribution function (PDF) transformation method, CFD theory and statistical probability distribution theory, which will be simply introduced in following section. It can be seen from the above analysis that the degradation reliability revolves around the information in degradation process, and it is also a combination of traditional reliability and degradation performance, which ultimately may contribute to the

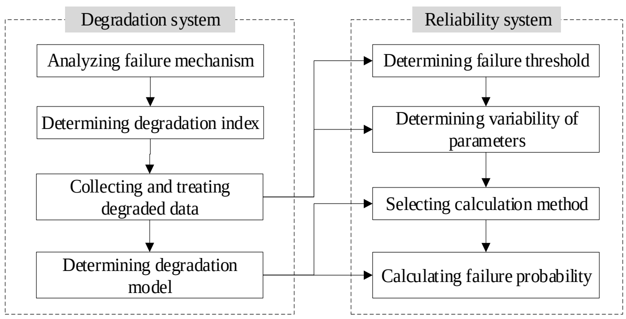

Fig. 1 Methodological framework for a coupling analysis of reliability in combination with degradation

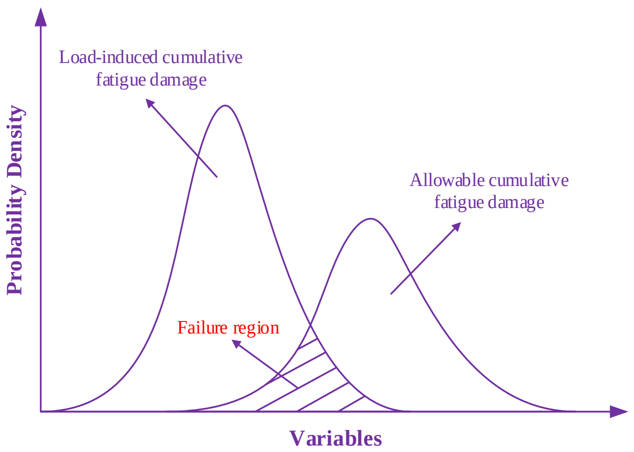

Fig. 2 Schematic diagram of probability distributions of load-induced and allowable CFD

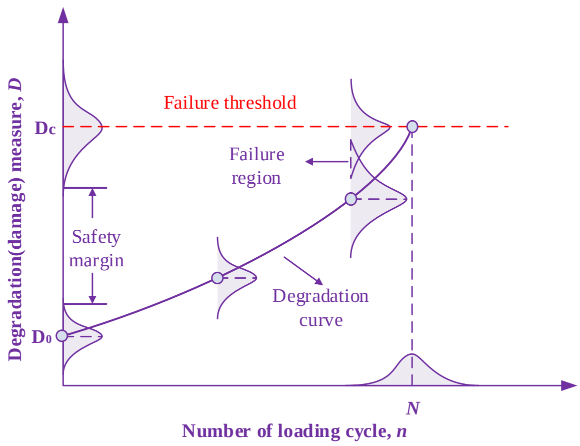

Fig. 3 Degradation performance (or CFD) profile under uncertainty

reduction of life-cycle cost in engineering.

The advantages of the reliability evaluation of pavement materials or structures using degradation data instead of failure time data are as follows. (1) Degradation of performance is a natural attribute of concrete pavements, and degradation data can be obtained by monitoring performance data whether or not the failure occurs; (2) Degradation data can be applied to situations where there is little or no failure and can provide more information than the failure time data (the reduction of the loss of failure time data); (3) Degradation data can provide more accurate life estimates than fatigue life tests with little or no failure. In other words, for no failure pavements, useful reliability can be extrapolated from the degradation data; (4) Since the modeling of degradation data is closer to the results of failure physics, degradation data can provide more information for the degradation process which can also contribute to the find of the physical relationship between the performance degradation and applied stress or time.

3. Key theories and techniques

3.1 Reliability theory

Various uncertain factors that affect the safety, applicability and durability of structures exist in the process of design, construction and service of engineering structures (Jia *et al*. 2018, Pei *et al*. 2016, Stubstad *et al*. 2002, Zhao *et al*. 2011). These uncertainties make it impossible to judge whether the structure is reliable by a simple 'yes' or 'no', but it must be described by a reliability index in terms of probability. In such a context, the reliability theory comes into being. It is the base of the assessment of the life-cycle performance of an engineering structure. Many literatures defined reliability as the probability that a structure is able to complete a required function within a given period under specified conditions (GB 50153-2008 2008, Pei *et al*. 2016, Zhao *et al*. 2011). Meanwhile, several limit state functions have been developed to analyze the structural reliability (Su *et al*. 2015, Zhao *et al*. 2011, Zhao *et al*. 2016). One of the most basic functions is the stress-strength (load-capacity) interference model, which is given by

$$G = x_S - x_F \tag{1}$$

where G denotes limit state function; x_S represents the applied load or load-induced response, i.e., 'stress'; x_F denotes the capacity of structures to withstand the applied 'stress' or an allowable quantity, i.e., 'strength'.

In the field of pavements, one of the concepts of the reliability is the probability that the quantity of pavement distress accumulated during desired service life does not exceed an allowable level (Pei *et al.* 2016, Zhang *et al.* 2018). Correspondingly, as for fatigue distress, x_S comes from pavement distress and is represented in terms of the CFD of pavement materials. x_F is denoted in terms of its allowable level during life-cycle period. Thus, generalized stress-strength interference model for pavements can be obtained, a schematic diagram of which is illustrated in Fig. 2. It can be seen from Fig. 2 that a failure occurs when the level of load-induced CFD exceeds that of allowable quantity, and the failure probability can be obtained according to the area of failure region.

This study adopts the generalized stress-strength interference model to analyze the degradation reliability of pavement materials. According to aforementioned analysis, the limit state function or safety margin $G(n)$ for fatigue distress can be given by

$$G(n) = \mathrm{D}_c - D_n \tag{2}$$

where n denotes number of loading cycles; D_n denotes the load-induced CFD at a given n; D_c denotes the critical or allowable CFD.

Dynamic variation of fatigue damage under uncertainty is illustrated in Fig. 3, where D_0 is the initial damage, usually equaling to zero, N is the number of loading cycles at fatigue failure (or fatigue life). It can be seen that the reliability implies that the degradation measure is varying probabilistically with number of loading cycle. As a measure of degradation, the CFD at any given n can be considered as a random variable following a probability distribution whose mean and variability vary with n. Since the CFD is a function of the number of loading cycle, given

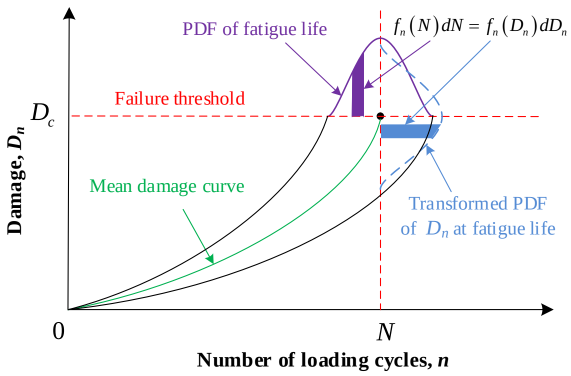

Fig. 4 Schematic view of PDF transformation of CFD

the model of D_n, the PDF of D_n could be obtained through the PDF of *N*. Generally, $\mathrm{G}(n) = 0$ denotes the limit state, $\mathrm{G}(n) > 0$ denotes the safety domain, and $\mathrm{G}(n) < 0$ denotes the failure domain. Cement concrete materials are thereby in a safety domain when D_n is at the range from 0 to D_c. The function of reliability (*R*) *vs* *n* can thereby be expressed by Eq. (3).

$$R = P_R\left(G(n) > 0\right) = \int_0^{D_c} f_n\left(D_n\right) dD_n \tag{3}$$

where $G(n)$ and $f_n(D_n)$ denote random variables being changed with *n*.

3.2 PDF transformation technique

One-to-one PDF transformation technique was constructed by Benjamin and Cornell (1970). According to the technique, the PDF of D_n at fatigue failure can be obtained due to the fatigue failure life is a discrete stochastic variable obeyed a specific distribution. Fig. 4 illuminates the scheme of the technique in this study.

Usually, the PDF of D_n at fatigue failure can be determined through the determining and differencing of the cumulative distribution function (CDF). According to the shade areas in Fig. 4, the transformation of the PDF of D_n at fatigue failure *vs* that of *N* is given by

$$f_n\left(D_n\right) dD_n = f_n\left(N\right) dN \tag{4}$$

3.3 Degradation model

In order to capture the degradation behavior of concrete materials, fatigue damage was adopted as a degradation indicator in this study because it is one of the primary failures of concrete pavement. To our knowledge, the CFD theories include two forms: linear and nonlinear, so the degradation model adopted in this study should be analysed based on the two theories respectively.

Regarding linear damage theories, the simplest and common method is Miner's rule (Miner 1945). The characteristic of the method has been studied extensively and deeply, and a great deal of test data have been accumulated. Numerous studies pointed out that the value of critical CFD equals 1 (Miner 1945, Ou 2016). These advantages contribute to the reliability evaluation and offer convenience for related engineering application. This theory can be expressed in the following equation.

$$D = n/N \tag{5}$$

where *D* denotes the damage characterized in terms of the cycle ratio.

However, numerous previous studies indicated that the accumulation of fatigue damage in concrete exhibits an obvious nonlinear characteristic (Bache and Vinding 1990, Darter 1990, Oh 1991). To our knowledge, the nonlinear damage theories have made great progress in the past, which mainly include the CFD models proposed by Marco *et al.* (1954), Henry (1955), Corten and Dolan (1956), Subramanyan (1976) and Shah (1984), respectively. These models are however difficult to be applied in practice because of its complexity and the difficulty of determination of parameters. Compared with the above models, the CFD models based on the irreversible thermodynamics and the continuum damage mechanics theory are more rigorous, which have a more distinct mathematical concept and more wide research and application prospects. One of the most representative models is the CFD theory proposed by Chaboche and Lesne (1988). This theory assumes that the fatigue damage of materials is related to the strain in internal plastic region, and the damage increment of each cycle is described by

$$\frac{\delta D}{\delta n} = \left[1 - (1 - D)^{1+p}\right]^q \left[\frac{\Delta\sigma}{M(1 - D)}\right]^p \tag{6}$$

$$q = q(\Delta\sigma) \tag{7}$$

$$\sigma = M(\sigma_m) \tag{8}$$

where *q*, *p* and *M* are constants related to temperature; $\Delta\sigma$ is a stress amplitude; σ_m is a mean stress. In general, *q* can be set to 0 for cement concrete materials. Based on Eq. (6), the CFD model can be derived as

$$D = 1 - \left(1 - \frac{n}{N}\right)^{\frac{1}{1+p}} \tag{9}$$

Considering that the Chaboche model (i.e. Eq. (9)) is consistent with the fatigue failure mechanism of materials, it enables the prediction of the residual life and strength more realistically. In this model, the parameter *p* is associated with stress ratio *r* and stress level *S*. A famous fatigue model considering the effect of stress ratio was introduced by Aas-Jakobsen (1970), which is given by

$$S = 1 - (1 - r)\beta \lg(N) \tag{10}$$

where, β denotes test parameter, which can be obtained by fitting fatigue test data. According to Eq. (10), Zhao and Sun (1999) derived and revised the CFD model based on the variation of actual stress level, and the obtained CFD model was shown in Eq. (11).

$$D = 1 - \frac{1}{\left[1 - \frac{(1 - r)\beta}{S} \lg\left(1 - \frac{n}{N}\right)\right]^2} \tag{11}$$

3.4 Weibull distribution theory

Fatigue behavior of concrete materials is often stochastic due to the randomness in fatigue resistance and that of the applied loads. Even though the amplitude is constant for fatigue test at any given stress levels, the fatigue life manifests as stochastic behavior with a specific distribution. So the fatigue life was described through statistical probability distribution in the past. The probability models describing the dispersion of fatigue failure life data mainly contain three models: normal distribution model, logarithmic normal distribution model and Weibull distribution model in which the two-parameter Weibull distribution (TPWD) is the most popular and widely used model in investigating on fatigue behaviour (Singh and Kaushik 2003, Zhu *et al*. 2017). In this study, the TPWD was utilized to analyze the stochastic fatigue life of concrete, the PDF and CDF of which are given by

$$f\left(N_i\right)=\frac{\alpha}{u}\left(\frac{N_i}{u}\right)^{\alpha-1}\exp\left[-\left(\frac{N_i}{u}\right)^{\alpha}\right] \tag{12}$$

$$F\left(N_i\right)=1-\exp\left[-\left(\frac{N_i}{u}\right)^{\alpha}\right] \tag{13}$$

where N_i denotes specific value of the random fatigue life; α denotes shape parameter at certain stress level; u denotes scale parameter or characteristic life at a certain stress level.

In this study, the TPWD can be verified through the graphical method. Thus, the survivorship function $L_N(N_i)$ of TPWD can be expressed by

$$L_N\left(N_i\right)=\exp\left[-\left(\frac{N_i}{u}\right)^{\alpha}\right] \tag{14}$$

Then

$$\ln\left[\ln\left(\frac{1}{L_N}\right)\right]=\alpha\ln N_i-\alpha\ln u \tag{15}$$

Eq. (15) indicates that $\ln[\ln(1/L_N)]$ changes linearly with $\ln(N_i)$. To obtain a graph from Eq. (15), the fatigue life data of concrete tested in this study should be scheduled in ascending order, and then the empirical survivorship function L_N is calculated through Eq. (16):

$$L_N=1-\frac{i}{k+1} \tag{16}$$

where i denotes failure order number; k represents total number of fatigue failure life data (or sample size) points at a given S. Specific analysis refers to the following section 6.

4. Calculation method for degradation reliability

As is well known, direct integration method (DIM), first order second moment method (FOSM) and Monte Carlo simulation method (MCSM) are the most commonly used methods to compute reliability. However, various methods have different limitations in application for reliability (Pei *et al.* 2016). DIM is a most accurate and basic method of computing reliability. It is, however, inconvenient in application for many complicated scenarios. FOSM method is widely used in engineering field because of its clear concept and simplicity. However, higher the nonlinearity of limit state function is, greater errors will. Hence, it fails to solve the nonlinear model-based reliability. In addition, the PDF of D_n at any given n is required for DIM and FOSM, but it fails to be directly computed in this study.

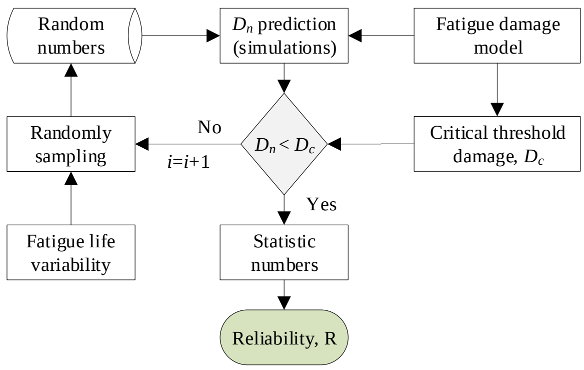

Fig. 5 Implementation scheme of the proposed method

At present, MCSM has become a primary mean to compute reliability. Its basic principle is that the failure probability is computed through the statistical theory (Rubinstein and Kroese 2007). The ability to deal with nonlinearity questions, more accurate and reliable, and direct solving are the advantages of this method. Generally, reliability can be obtained through MSCM when limit state function and PDF of random independent variables are given. Thus, a novel simulation method is proposed based on this in this study. An implementation scheme of the method is shown in Fig. 5. The expression to estimate degradation reliability and the variance are as follows.

$$\begin{aligned}R&=P\left(0<D_n\le D_c\right)\\&=\frac{\text{number of simulations within given range}}{\text{total number of simulations}}\end{aligned} \tag{17}$$

$$\sigma^2=\frac{R\left(1-R\right)}{\text{total number of simulations}} \tag{18}$$

5. Experiment

Ordinary portland cement, pebbles coarse aggregates (maximum size 20 mm) and natural sand were used. The achieved water-cement ratio, sand-cement proportion and coarse aggregate-cement proportion are 0.40%, 1.16% and 2.47% respectively. All the mixtures were mixed in the mixing plant. The specimens were cast in batches, each batch consists of 100 mm×100 mm×515 mm standard beam specimens and 100 mm×100 mm×100 mm cube specimens. To check the quality of each batch of concrete, the 28-days compression strength tests conducted on the cube specimens. The average (AVG) and coefficient of variation (COV) of the obtained 28-days compression strengths were 57.14 MPa and 6.8% respectively. Static flexural tests were

Table 1 Laboratory fatigue life data for cement concrete

Specimen number	Stress levels		
	0.675	0.750	0.800
1	89063	10003	2110
2	90125	10943	2275
3	92940	11882	2659
4	114369	12413	3105
5	115684	14762	3758
6	116040	19048	4165
7	121427	20284	4749
8	168803	20328	5073
9	172933	23286	5981
10	175168	26291	6844
11	193110	26539	7161
12	193830	26561	8027
13	209442	40095	8345
14	258418	53683	8411
15	-	58835	-

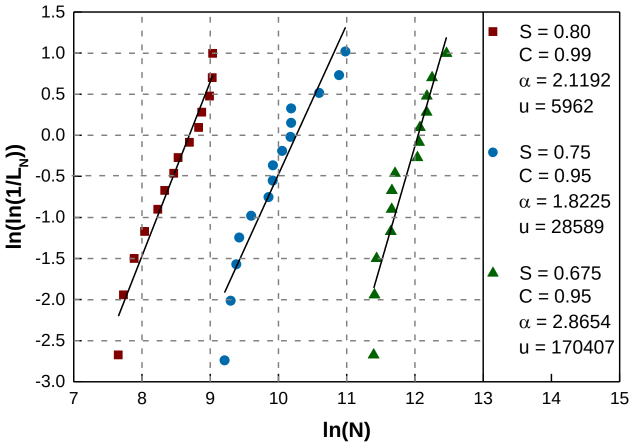

Fig. 6 Graphical analysis of fatigue life data of cement concrete at different stress levels

performed on the beam specimens in order to determine the applied minimum and maximum load limits for flexural fatigue tests. The tests were conducted under a three-point loading configuration. The specimens for the tests were installed in a 100kN Materials Test System (MTS) closed-loop electrohydraulic testing system. The AVG and COV of the obtained static flexural strengths were 11.59 MPa and 9.29% respectively.

The remaining specimens were used to conduct flexural fatigue tests. The span/points of loading in the tests were the same as in the static flexural tests. Flexural fatigue tests were performed at a constant amplitude with a sinusoidal load pulse and a frequency of 10 Hz for different stress levels (S), ranging from 0.80 to 0.675. The stress ratio (r) was set to 0.10 during the tests. Details of the test and the raw results can be also found in the previous literature (Li and Che 1998). The test was terminated when the beam specimen occurs failure. The flexural fatigue results are presented in Table 1.

6. Results and discussion

6.1 Fatigue failure life distribution

According to the flexural fatigue test results in this study, the Weibull parameters for cement concrete at different stress levels obtained are shown in Tables 2-4. Fig. 6 shows the fitting diagrams of the Eq. (15) and the treated test data in this study at different stress levels. It can be seen that the correlation coefficient, C, is 0.99, 0.95 and 0.95 at stress levels 0.80, 0.75 and 0.675 respectively, implying that the TPWD enables the modeling of fatigue life data for cement concrete in this study. Fig. 6 also gives the values of Weibull parameters. The specific method to determine the two parameters is as follows: the shape parameter α was estimated by the slope of the line in the figure, and the fatigue life N corresponding to $L_N = 0.368$ or $P_f = 0.6320$ provided an estimate of the scale parameter u. In addition, the two parameters were also estimated directly through the regression analysis. The parameters estimated for three stress levels are α=2.1192 and u=5962 for S=0.80, α=1.8225 and u=28589 for S=0.75, and α=2.8654 and u=170407 for S=0.675. The dispersion of fatigue life data allows to be described in terms of the scale parameters to some extent, that is, the larger scale parameters correspond

Table 2 Fatigue life and survivorship function at stress level S=0.675 for cement concrete

Stress Level	i	N_i	$L_N = 1-(i/(k+1))$	$\ln[\ln(1/L_N)]$	$\ln(N_i)$	$P_f = 1 - L_N$
0.675	1	89063	0.9333	-2.6738	11.3971	0.0667
	2	90125	0.8667	-1.9442	11.4090	0.1333
	3	92940	0.8000	-1.4999	11.4397	0.2000
	4	114369	0.7333	-1.1707	11.6472	0.2667
	5	115684	0.6667	-0.9027	11.6586	0.3333
	6	116040	0.6000	-0.6717	11.6617	0.4000
	7	121427	0.5333	-0.4642	11.7071	0.4667
	8	168803	0.4667	-0.2716	12.0365	0.5333
	9	172933	0.4000	-0.0874	12.0607	0.6000
	10	175168	0.3333	0.0940	12.0735	0.6667
	11	193110	0.2667	0.2790	12.1710	0.7333
	12	193830	0.2000	0.4759	12.1747	0.8000
	13	209442	0.1333	0.7006	12.2522	0.8667
	14	258418	0.0667	0.9962	12.4623	0.9333

Table 3 Fatigue life and survivorship function at stress level *S*=0.750 for cement concrete

Stress level	i	N_i	$L_N = 1-(i/(k+1))$	$\ln[\ln(1/L_N)]$	$\ln(N_i)$	$P_f = 1-L_N$
0.750	1	10003	0.9375	-2.7405	9.2106	0.0625
	2	10943	0.8750	-2.0134	9.3005	0.1250
	3	11882	0.8125	-1.5720	9.3828	0.1875
	4	12413	0.7500	-1.2459	9.4265	0.2500
	5	14762	0.6875	-0.9816	9.5998	0.3125
	6	19048	0.6250	-0.7550	9.8547	0.3750
	7	20284	0.5625	-0.5528	9.9176	0.4375
	8	20328	0.5000	-0.3665	9.9198	0.5000
	9	23286	0.4375	-0.1903	10.0556	0.5625
	10	26291	0.3750	-0.0194	10.1770	0.6250
	11	26539	0.3125	0.1511	10.1864	0.6875
	12	26561	0.2500	0.3266	10.1872	0.7500
	13	40095	0.1875	0.5152	10.5990	0.8125
	14	53683	0.1250	0.7321	10.8909	0.8750
	15	58835	0.0625	1.0198	10.9825	0.9375

Table 4 Fatigue life and survivorship function at stress level *S*=0.800 for cement concrete

Stress level	i	N_i	$L_N = 1-(i/(k+1))$	$\ln[\ln(1/L_N)]$	$\ln(N_i)$	$P_f = 1-L_N$
0.800	1	2110	0.9333	-2.6738	7.6544	0.0667
	2	2275	0.8667	-1.9442	7.7297	0.1333
	3	2659	0.8000	-1.4999	7.8857	0.2000
	4	3105	0.7333	-1.1707	8.0408	0.2667
	5	3758	0.6667	-0.9027	8.2316	0.3333
	6	4165	0.6000	-0.6717	8.3345	0.4000
	7	4749	0.5333	-0.4642	8.4657	0.4667
	8	5073	0.4667	-0.2716	8.5317	0.5333
	9	5981	0.4000	-0.0874	8.6963	0.6000
	10	6844	0.3333	0.0940	8.8311	0.6667
	11	7161	0.2667	0.2790	8.8764	0.7333
	12	8027	0.2000	0.4759	8.9906	0.8000
	13	8345	0.1333	0.7006	9.0294	0.8667
	14	8411	0.0667	0.9962	9.0373	0.9333

Table 5 K-S test results at stress level *S*=0.675 for cement concrete

Stress level	i	x_i	$F^*(x_i) = i/k$	$F_N(x_i)$	$\lvert F^* - F \rvert$
0.675	1	89063	0.0714	0.1443	0.0728
	2	90125	0.1429	0.1489	0.0060
	3	92940	0.2143	0.1614	0.0529
	4	114369	0.2857	0.2731	0.0126
	5	115684	0.3571	0.2808	0.0763
	6	116040	0.4286	0.2829	0.1457
	7	121427	0.5000	0.3152	0.1848
	8	168803	0.5714	0.6222	0.0507
	9	172933	0.6429	0.6476	0.0048
	10	175168	0.7143	0.6611	0.0531
	11	193110	0.7857	0.7609	0.0248
	12	193830	0.8571	0.7646	0.0926
	13	209442	0.9286	0.8357	0.0929
	14	258418	1.0000	0.9630	0.0370

Note: $T_1 = 0.1848$, $T_c = 0.35$ (5% significance level), $T_1 < T_c$ (accepted)

to the more dispersed fatigue life data. So it is noted that the dispersion of fatigue life for concrete decreases with the increase of stress level in this study.

6.2 Goodness-of-test for fatigue life

The above analysis indicates that the TPWD describes well the fatigue life data in this study, and its shape and scale parameters have been obtained through graphical method. In addition to the use of the graphical method, the goodness-of-fit test would be far more convincing to reveal that the TPWD is an available model to capture fatigue life data of concrete in this study.

The Kolmogorov-Smirnov (K-S) test is a nonparametric goodness-of-fit test and used to determine whether a potential distribution is consistent with a hypothesized probability distribution **(**Dodge 2008**)**. One the advantage of the K-S test is that it is able to take into account the distribution functions jointly. In this case, one random sample can be only obtained from fatigue life data which obeys the specific and known distribution function. The K-S test can be performed through the Eqs. (19)-(20)

$$T_1 = \sup_{x_i} \left| F^*(x_i) - F_N(x_i) \right| \tag{19}$$

$$F^*(x_i) = i/k \tag{20}$$

where, $F_N(x_i)$ denotes hypothesized CDF given by Eq.

Table 6 K-S test results at stress level *S*=0.750 for cement concrete

Stress level	i	x_i	$F^*(x_i) = i/k$	$F_N(x_i)$	$\|F^* - F\|$
0.750	1	10003	0.0667	0.1371	0.0705
	2	10943	0.1333	0.1595	0.0262
	3	11882	0.2000	0.1828	0.0172
	4	12413	0.2667	0.1964	0.0703
	5	14762	0.3333	0.2590	0.0743
	6	19048	0.4000	0.3794	0.0206
	7	20284	0.4667	0.4143	0.0523
	8	20328	0.5333	0.4156	0.1178
	9	23286	0.6000	0.4974	0.1026
	10	26291	0.6667	0.5761	0.0905
	11	26539	0.7333	0.5824	0.1510
	12	26561	0.8000	0.5829	0.2171
	13	40095	0.8667	0.8431	0.0235
	14	53683	0.9333	0.9573	0.0239
	15	58835	1.0000	0.9759	0.0241

Note: $T_1 = 0.2171$, $T_c = 0.34$ (5% significance level), $T_1 < T_c$ (accepted)

Table 7 K-S test results at stress level *S*=0.800 for cement concrete

Stress level	i	x_i	$F^*(x_i) = i/k$	$F_N(x_i)$	$\|F^* - F\|$
0.800	1	2110	0.0714	0.1048	0.0333
	2	2275	0.1429	0.1217	0.0211
	3	2659	0.2143	0.1653	0.0490
	4	3105	0.2857	0.2219	0.0638
	5	3758	0.3571	0.3134	0.0437
	6	4165	0.4286	0.3735	0.0551
	7	4749	0.5000	0.4607	0.0393
	8	5073	0.5714	0.5085	0.0630
	9	5981	0.6429	0.6346	0.0083
	10	6844	0.7143	0.7381	0.0238
	11	7161	0.7857	0.7711	0.0146
	12	8027	0.8571	0.8471	0.0100
	13	8345	0.9286	0.8699	0.0587
	14	8411	1.0000	0.8743	0.1257

Note: $T_1 = 0.1257$, $T_c = 0.35$ (5% significance level), $T_1 < T_c$ (accepted)

(13).

Table 5 shows the results obtained by the K-S test for *S*=0.80 for concrete in this study. It can be seen that the greatest vertical distance, T_1, is 0.1257 for this stress level. The critical value T_c for *k*=14 and 5% significance level is 0.35 from the K-S table. Considering $T_1 < T_c$ (0.1257<0.35), the TPWD must be accepted at 5% significance level, for the fatigue life at stress level of 0.80. The K-S test was also conducted on the fatigue life data at all other stress levels (0.75 and 0.675) in this study, as shown in Tables 6-7. It was seen from the two tables that the greatest vertical distance lower than the critical value, that is, 0.2171<0.34 for *S*=0.75 and 0.1848<0.35 for *S*=0.675, which indicates that the TPWD also was acceptable at 5% significance level for the two stress levels of 0.75 and 0.675. Above all, the parameters obtained by the graphical method can satisfy the requirement of the analysis of the random characteristic of flexural fatigue life of concrete.

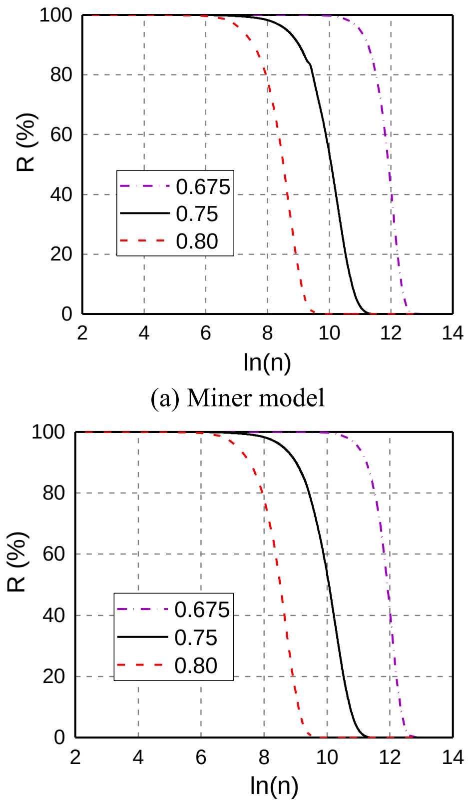

Fig. 7 The evolution of reliability of plain cement concrete for pavement during flexural fatigue testing

6.3 Reliability evaluation

This study ultimately aims to assess degradation reliability of plain concrete for pavement under flexural fatigue loading. According to the steps in Fig. 5, the degradation reliability under different stress levels was obtained. Since the degradation model plays an important role in degradation reliability modeling, the degradation models should be first determined. Fatigue damage is divided into two kinds linear damage and nonlinear damage, so Miner model and Chaboche model were adopted based on the section 3.3 in this study. For the fatigue strength of concrete under flexural fatigue loading in this study, the fatigue test data was fitting using Eq. (10), and the test parameter β is 0.0658. On the basis of the tested data in this study, the values between loading cycles and CFD were obtained using Eq. (11) at different stress levels respectively. These values can be regressed through the Chaboche model, and thus p was obtained, which is 12.6069, 14.0685 and 15.0401 for the stress levels of 0.675, 0.75 and 0.80, respectively. Moreover, the fatigue life data obeys the TPWD, the distribution parameters are shown in Fig. 6. The random numbers of D_n can be generated based on the two CFD models and the distribution of *N*. The critical threshold damage (D_c) of the two models is 1. The degradation reliability of plain cement concrete for pavement are shown in Fig. 7.

It can be found that the reliability decreases with an increase in number of loading cycles for a particular stress level *S*. Also, the evolution of reliability for concrete materials under flexural fatigue loading consists of three stages: high and stable stage (100%), rapidly reducing stage and low and stable stage (0%). This may be explained by the propagation of cracks due to flexural fatigue loads. The high and stable stage may be associated with the initiation of cracks in concrete, the secondary stage may correspond to the propagation of cracks in concrete, and the tertiary stage may correspond to the fatigue failure of concrete. It can be also found that the presence of higher stress levels *S*, the periods of crack initiation for concrete materials are shorter and fatigue failure earlier occurs. With the decrease of degradation rate of reliability, the fatigue damage for plain concrete will accumulated at a higher rate, implying that the higher stress level causes the lower reliability for plain concrete during the reducing stage. In addition, it can be also seen that the reliability for average fatigue life is in the secondary stage and about equals to 50%. In practical pavement project, the reliability of plain cement concrete slabs should thereby be more than 50% as far as possible.

In summary, the methodological framework constructed by this study makes it possible to use an information in a degradation process, compute the degradation reliability of plain concrete for pavement under flexural fatigue loading and describe its more realistic CFD behavior. The proposed computing method may be extended to stochastic damage accumulation analysis under non-fatigue degradation behaviors.

7. Conclusions

The following conclusions may be drawn from the present study:

- The constructed methodological framework analyzing the reliability made it possible to use an information in a degradation process, enabling the analysis of the reliability of plain concrete for pavement and the description of its more realistic CFD behavior.
- The dispersion of flexural fatigue life for concrete in this study decreased with the increase of applied stresslevel. The flexural fatigue life of concrete at different stress levels obeyed the TPWD.
- The proposed new calculation method combined four theories and techniques, including reliability theory, PDF transformation method, CFD theory, statistical probability distribution theory and MCSM, which treated degradation as a CFD phenomenon and might also be extended to damage accumulation analysis under non-fatigue degradation behaviors.
- The evolution of reliability for tested materials in this study was divided into three stages. This might be explained by the propagation of cracks due to fatigue loads. Each stage might correspond to the development of cracks to some extent. The reliability at the average fatigue life data was in the secondary or rapidly reducing stage for all stress levels. In practical project, the reliability of cement concrete slabs should thereby be more than 50% as far as possible.

The established methodological framework is generic and also is able to analyze the reliability of other materials. Extensive work on the calculation method proposed in this study is still need when handling other degradation models, such as pavement performance models, or other uncertainties, such as freeze-thaw action. In addition, the methods and findings of this study may be serving the purpose for balancing the safety and risk cost for concrete slabs under an uncertain action.

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