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Numerical study for nonlocal vibration of orthotropic SWCNTs based on Kelvin's model

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Abstract. This research deals with the study of the orthotropic vibrational features of single-walled carbon nanotubes according to Kelvin's model and to check the accuracy of the models, the results have been compared with earlier modeling/simulations. Obtaining rough approximations of the natural frequencies of CNTs using continuum equations are still a common procedure, even at high harmonics. The effects of different physical and material parameters on the fundamental frequencies are investigated for zigzag and chiral single-walled carbon nanotubes invoking Kelvin's theory. By using nonlocal Kelvin's model, the fundamental natural frequency spectra for two forms of single-walled carbon nanotubes (SWCNTs) have been calculated. The influence of frequencies with nonlocal parameters and bending rigidity are investigated in detail for these tubes. Computer software MATLAB is utilized for the frequencies of SWCNTs and current results shows a good stability with comparison of other studies.

Keywords: SWCNTs; wave propagation approach; Kelvin Model; bending rigidity

1. Introduction

Carbon nanotube as one of the most applicable miniature structure attracts many researchers in order to analytically and experimentally probes its dynamical properties using the nonlocal beam theory. Several researchers studied linear and nonlinear vibrations of the nanostructures utilizing the Eringen's nonlocal elasticity theory Eringen (2002). They mostly focused on the free vibrational analysis of the nano-structure, specially, carbon nanotubes. In addition, nano-structures can be mentioned as the important types of devices which have wide applications in a variety of technological and scientific fields. The nonlinear forced vibration of carbon nanotubes has seldom been observed. However, this issue is very crucial due to the widespread application of the forced nonlinear vibration carbon nanotubes in many practical instruments.

The nonlocal elasticity introduced by Eringen (2002) becomes a turning point as small scale effect was inculcated in to fundamental equations as simply material parameter. Therefore, scientific community now propose to apply nonlocal continuum models to investigate nano-structured materials (Sudak 2003, Wang *et al*. 2006, Pradhan and Phadikar 2009, Ansari *et al*. 2010, Hao *et al*. 2010, Amara *et al.* 2010, Shen and Zhang 2010). Donnell (1996), Flügge (1962) have been two substantial shell theories practiced extensively in study of static and dynamic characteristics of CNTs. Flügge shell theory takes promising place to generate remarkably accurate developments to examine the CNTs. The existence of long range interactions in materials is the basic reason of application of nonlocal theory. The first ever work presented on use of nonlocal elasticity was by Peddieson *et al*. (2003). Prominent computational competence and accuracy makes nonlocal models an attractive choice for further advancements in field. Wang *et al.* (2006) introduced new modeling for vibration of CNTs and to find the critical buckling strain and tube thickness. Natuski *et al.* (2007) investigated single and double-walled CNTs filled with fluids by adopting wave propagation approach. Flügge shell theory was proposed to form governing equations of motion for CNTs. Lee and Chang (2008) analyzed the vibration mode shape and frequency of fluid-filled SWCNTs. It is found that mode shape and frequency are influenced significantly by the nonlocal parameters. Ke *et al.* (2009) investigated free nonlinear vibrations of double-walled CNT and applied differential quadrature technique to derive frequency equations. On the other side, for length scale coefficient and soft elastic medium with embedded carbon nanotube, the nonlocal frequencies are comparatively lower. It is also found that the frequencies of the nonlocal model at different stages of temperature are higher than the nonlocal with same temperature. Eringen nonlocal theory and Von-Karman geometry were fully studied by Yang *et al.* (2010). Selmi and Bisharat (2018) studied the Aluminum alloy (Al-alloy) reinforced with Single walled carbon nanotubes (SWNT), which represents an important industrial application.

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Different beam theories (BT) are applied to investigate functionally graded (FG) beams made of Al-alloy reinforced with randomly oriented, straight and long SWNT. The Rayleigh-Ritz method is used to estimate the beam frequencies. Bisen *et al*. (2018) investigated the the natural fiber (Luffa cylindrica fiber) reinforced epoxy composite and their structural responses (frequency and deflection) have been computed experimentally and numerically first time using the corresponding experimental elastic properties. Selmi (2019) investigated the effectiveness of single walled carbon nanotubes in improving the dynamic behavior of cracked Aluminium alloy, Al-alloy, beams by using a method based on changes in modal strain energy. Mechanical properties of composite materials are estimated by the Eshelby-Mori-Tanaka method.

Rouhi and Ansari (2012) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon nonlocal Flügge shell theory. Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for nonlocal parameter. Ansari and Rouhi (2013) summarized the effect of small scale, geometrical parameter and layer-wise end conditions of double-walled CNT by adopting Flügge shell model (FSM). They depicted that the continuum model considering the nonlocal effect compels the short double-walled CNT more flexible. Mehar and Panda (2018a) computed the vibration behavior, bending and dynamic response of FG reinforced CNT using shear deformation theory and finite element method. For the sake of generality, the mathematical model was presented with the mixture of Green Lagrange method. The convergence of these methodologies have been checked for the variety of results. The composite plates with different graded was investigated with isotropic and core phase. Moreover, Benguidiab *et al.* (2014) explored the features of zigzag double-walled CNT. A comprehensive research presented by Salvatore Brischetto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. Hussain and Naeem (2017) examined the frequencies of armchair tubes using Flügge's shell model. The effect of length and thickness-to-radius ratios against fundamental natural frequency with different indices of armchair tube was investigated. Dihaj *et al*. (2018) studied the transverse free vibration of chiral double-walled carbon nanotube (DWCNTs) embedded in elastic medium by the non-local elasticity theory and Euler Bernoulli beam model. The governing equations are derived and the solutions of frequency are obtained. Hussain and Naeem (2018a) used Donnell's shell model to calculate the dimensionless frequencies for two types of single-walled carbon nanotubes. The frequency influence was observed with different parameters. Fatahi-Vajari *et al*. (2019) studied the vibration of single-walled carbon nanotubes based on Galerkin's and homotopy method. This work analyses the nonlinear coupled axial-torsional vibration of single-walled carbon nanotubes (SWCNTs) based on numerical methods. Two-second order partial differential equations that govern the nonlinear coupled axial-torsional vibration for such nanotube wasderived. Asghar *et al*. (2019a, b) conducted the vibration of nonlocal effect for double-walled carbon nanotubes using wave propagation approach. Many material parameters are varied for the exact frequencies of many indices of double-walled carbon nanotubes.

Shahrma *et al*. (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Arani *et al.* (2016) used the nonlinear buckling of SWCNTs and the mixture rule was employed for buckling analysis of embedded CNTs with Euler and Timoshenko beam model. The influence of geometrical parameter and elastic foundation with different boundary conditions was investigated. Hussain *et al*. (2017) demonstrated an overview of Donnell theory for the frequency characteristics of two types of SWCNTs. Fundamental frequencies with different parameters have been investigated with wave propagation approach. Chemi *et al*. (2018) determined the nonlocal critical buckling loads of chiral double-walled carbon nanotubes embedded in an elastic medium, the nonlocal Timoshenko beam theory. The solution for the nonlocal critical buckling loads is obtained using governing equations of the nonlocal theory. Mehar *et al*. (2017a, b) studied the frequency response of FG CNT and reinforced CNT using the simple deformation theory, finite element modeling, Mori-Tanaka scheme. They investigated a new frequency phenomena with the combination of Lagrange strain, Green-Lagrange, for double curved and curved panel of FG and reinforced FG CNT. The charactrictics of sandwich and grades CNT was found with labeling the temperarure environ. The thermoelastic frequency of single shaollow panel was determined using Mori-Tanake formaulation. The research of these authors have opened a new frequency spectra for other material researchers. Salah *et al*. (2019) examined a simple four-variable integral plate theory to investigate thethermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Bilouei *et al.* (2016) and Zamanian *et al.* (2017) studied the buckling behavior of concrete columns with nanofiber reinforced polymer and SiO_2 nano-particles. By using the strain-displacements, Hamilton's principles and Mori- Tanka approach, the governing equation was derived. Numerical results were presented with the variation of elastic foundations. Narwariya *et al*. (2018) presented the vibration and harmonic analysis of orthotropic laminated composite plate. The response of plate is determined using Finite Element Method. The eight noded shell 281 elements are used to analyze the orthotropic plates and results are obtained so that the right choice can be made in applications such as aircrafts, rockets, missiles, etc. to reduce the vibration amplitudes. Mehar *et al*. (2018) evaluated the frequency behavior of nanolpate structure using FEM including the nonlocal theory of elasticity. Computer generated results are created by using the software first time roubustly to check the vibration of

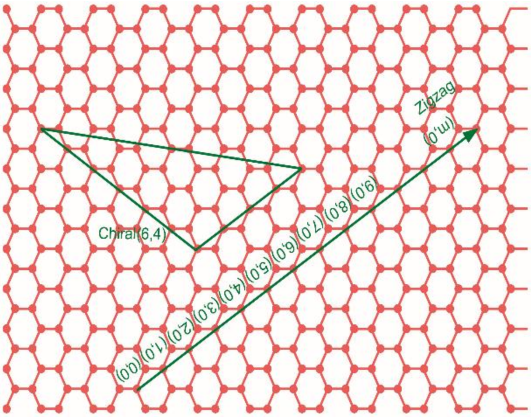


Fig. 1 Hexagonally description of zigzag and chiral SWCNTs on the graphene sheet

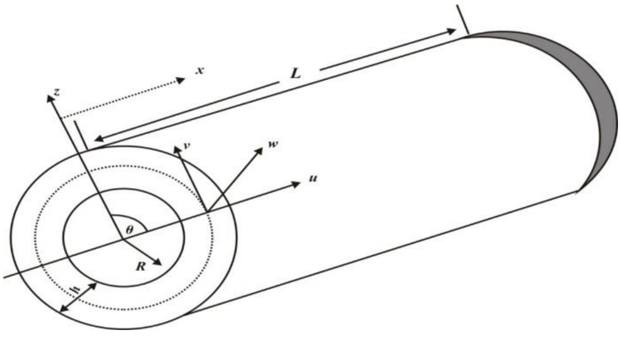


Fig. 2 Geometry of SWCNTs.

nanoplate. The efficiency was checked by comparing the results of available data. Behera and Kumari (2018) conducted first time, an exact solution for free vibration of the Levy-type rectangular laminated plate considering the most efficient Zig-Zag theory (ZIGT) and third order theory (TOT). The plate is subjected to hard simply supported boundary condition (Levy-type) along x axis. Yazid *et al.* (2018) presented new refined plate theory by employing nonlocal small effects. By using the principle of virtual displacements, the nonlocal relation for equation of motion was obtained. The results presented here may provide a useful design for nanostructures. In another study the viscoelastic effects of the medium were also studied using Kelvin model for the medium surrounding microtubules (MTs) but for the MTs the same classical orthotropic elastic shell model was used (Safeer *et al.* 2019). Mehar and Panda (2018b) investigated the curved shell and CNT vibration with thermal environment using higher order deformation theory. These CNT was mixed with different configurations of the layers. The results have been verified with the earlier investigations. Batou *et al*. (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new higher shear deformation theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. Many researchers directly used the classical theory for the structure of CNTs (He *et al.* 2005, Hu *et al.* 2007, Gibson *et al.* 2007, Ghavanloo *et al.* 2010, Yoon *et al.* 2002, Mehar *et al*. 2016, Mehar *et al*. (2019). The use of wave propagation approach is important for the study of nanostructures to develop a new formulism with different theories. In this approach, eigenvalue form is developed with the help of axial modal function in matrix representation. With the help of computer software MATLAB, frequencies of SWCNTs are extracted. The formulation of WPA is given by Zhang *et al.* (2001), a brief yet simple explanation first time. Recently Hussain and Naeem (2019a, b, c, d, 2020) and performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. They investigated many physical parameters for the rotating and non-rotating vibrations of armchair, zigzag and chiral indices. Moreover, the mass density effect of single walled carbon nanotubes with in-plane rigidity have been calculated for zigzag and chiral indices. Due to the exactness of this approach, some researchers have been used for the vibration of CNT/shells (Simsek 2010, Narendar 2011, Hussain and Naeem 2018).

In present paper, vibrations of SWCNTs for zigzag indices (9, 0), (16, 0), (21, 0) and chiral indices (13, 6), (23, 8), (26, 11) have been analyzed with specified conditions. We developed a new model from the combination of the nonlocal Kelvin's model. The governing equation has been developed for the vibrations of SWCNTs considering the nonlocal parameter. Effects of nonlocal parameters and bending rigidity were fully investigated on the fundamental natural frequency (FNF) against aspect ratios. It has been shown that frequency curves decrease as an increment in the nonlocal parameter and increases by increasing of the aspect ratio. Additionally, it can be seen that by increasing in-plane rigidity, the frequencies would increase. Also the frequency curves for C-F are lower throughout the computation than that of C-C curves.

2. Materials and methods

When a graphene sheet is rolled with its hexagonal cells, the structure can be conceptualized as SWCNTs and its circumference and quantum properties depend upon the chirality and diameter described as a pair of (n, m). In addition, the integer's n and m represent the orientation of the graphene honeycomb lattice. Fig. 1 shows the orientation of the graphene sheet as, the nanotubes are zigzag, if m=0; nanotubes become chiral, if $n \neq m$; the geometry of SWCNTs is shown in Fig. 2. We will apply nonlocal orthotropic elastic shell model to analyze the wave propagation of CNTs. Surrounding medium of CNTs will be modeled by Kelvin model. We will develop nonlocal orthotropic Kelvin-like model by the combination of these models. We will use wave propagation approach to find the wave dispersion relations for CNTs in viscoelastic medium.

2.1 Nonlocal orthotropic Kelvin-like model

Cemal Eringen are pioneers of the nonlocal theory (Kröner 1967). For an elastic and homogeneous material the stress strain relationships are given below

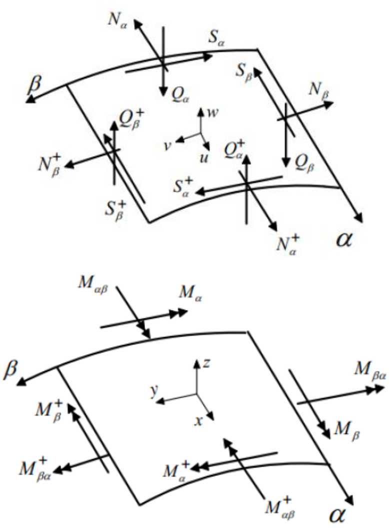


Fig. 3 Resolution of components of stress and moments of the middle surface of CNTs

$$\sigma_{ij,j} = 0 \tag{1}$$

$$\sigma_{ij}(x) = \int \varphi(|x' - x|, \psi) C_{ijkl} \varepsilon_{kl}(x') dV(x'), \ \forall\, x \in V \tag{2}$$

$$\varepsilon_{ij} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where j denotes the derivative with respect to j, σ_{ij} and ε_{kl} are strain tensor and stress tensor respectively, and elastic modulus tensor is denoted by C_{ijkl}, u_i represents the displacements, the attenuation function is $\varphi(|x' - x|, \tau)$, and $|x' - x|$ denotes the usual distance. Also, $\psi = e_0 a/l$, where e_0 is a material constant, internal characteristics length is represented by a and l denotes the external characteristics length.

The differential form of Eq. (2) is used as nonlocal constitutive relation (Eringen 2002)

$$(1 - (e_o a)^2 \nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{4}$$

where a is the internal characteristic length.

In this study we have taken $e_0 a$ as a single parameter, known as small scale parameter which represents the effect of size for the nano and micro structures, and ∇^2 is the Laplace operator. Our coordinate system x, y and z are axial, circumferential and radial coordinates respectively whose dimensionless coordinates are $\alpha = x/R$, $\beta = y/R$ and $\gamma = z/R$.

Along α, β and γ directions, the displacement of middle surface are u, v and w, respectively. The geometrical relations are given by Flugge's shell theory (Flugge 1973, Zou and Foster 1995, Paliwal *et al.* 1995)

$$\varepsilon_\alpha = \tfrac{1}{R}\left(\frac{\partial u}{\partial \alpha} - \gamma \frac{\partial^2 w}{\partial \alpha^2}\right) \tag{5}$$

$$\varepsilon_\beta = \tfrac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right) - \tfrac{\gamma}{R(1+\gamma)}\left(\frac{\partial^2 w}{\partial \beta^2} + w\right) \tag{6}$$

$$\varepsilon_{\alpha\beta} = \tfrac{\gamma}{R(1+\gamma)}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + 2\gamma\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha \partial \beta}\right) + \gamma^2\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha \partial \beta}\right)\right] \tag{7}$$

The stress-strain relationships in dimensionless coordinates derived from Eq. (4) is as under (Gao and An 2010)

$$\sigma_\alpha - (e_o a)^2 \nabla^2 \sigma_\alpha = E_1(\varepsilon_\alpha + \mu_1 \varepsilon_\beta)/(1 - \mu_1 \mu_2) \tag{8}$$

$$\sigma_\beta - (e_o a)^2 \nabla^2 \sigma_\beta = E_2(\varepsilon_\beta + \mu_2 \varepsilon_\alpha)/(1 - \mu_1 \mu_2) \tag{9}$$

$$\tau_{\alpha\beta} - (e_o a)^2 \nabla^2 \tau_{\alpha\beta} = G \varepsilon_{\alpha\beta} \tag{10}$$

where σ_α, σ_β and $\tau_{\alpha\beta}$ are normal and shear stresses, and ε_α, ε_β and $\varepsilon_{\alpha\beta}$ are respective strains; E_1 and E_2 are moduli of elasticity; Poisson's ratios in the directions of α and β are μ_2 and μ_1 respectively. G is modulus of rigidity or shear modulus. Also we have $E_1\mu_1 = E_2\mu_2$ and $\nabla^2 = (\partial^2/\partial\alpha^2 + \partial^2/\partial\beta^2)/R^2$ which is the Laplace operator in dimensionless coordinates. The element of tube in our coordinates is shown in Fig. 3, where (N, S, Q) are the stress resultants and (M) is the moment. The thermal expansion causes pre-stress, which is neglected because the present temperature is considered as the reference temperature. We arrive at the dynamic equilibrium equations

$$\begin{cases} \dfrac{\partial N_\alpha}{\partial \alpha} + \dfrac{\partial S_\beta}{\partial \beta} + \kappa = \rho h R \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial N_\beta}{\partial \beta} + \dfrac{\partial S_\alpha}{\partial \alpha} + Q_\beta = \rho h R \dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial Q_\alpha}{\partial \alpha} + \dfrac{\partial Q_\beta}{\partial \beta} + N_\beta = \rho h R \dfrac{\partial^2 w}{\partial t^2} \end{cases} \tag{11}$$

$$\begin{cases} \dfrac{\partial M_{\alpha\beta}}{\partial \alpha} + \dfrac{\partial M_\beta}{\partial \beta} - R Q_\beta = 0 \\ \dfrac{\partial M_{\beta\alpha}}{\partial \beta} + \dfrac{\partial M_\alpha}{\partial \alpha} - R Q_\alpha = 0 \end{cases} \tag{12}$$

where ρ is the mass density.

The resultants (N, S, Q) are derived from above set of integral equations using the stress components.

$$(1 - (e_0 a)^2 \nabla^2)\begin{bmatrix} N_\alpha, S_\alpha, \\ M_\alpha, M_{\alpha\beta} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_\alpha, \tau_{\alpha\beta}, \\ z\,\sigma_\alpha, z\,\tau_{\alpha\beta} \end{bmatrix}\left(1 + \frac{z}{R}\right) dz \tag{13}$$

$$(1 - (e_0 a)^2 \nabla^2)\begin{bmatrix} N_\beta, S_\beta, \\ M_\beta, M_{\beta\alpha} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_\beta, \tau_{\beta\alpha}, \\ z\,\sigma_\beta, z\,\tau_{\beta\alpha} \end{bmatrix} dz \tag{14}$$

$$(1 - (e_0 a)^2 \nabla^2)\left(Q_\alpha, Q_\beta\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\tau_{\alpha z}, \tau_{\beta z}\right] dz \tag{15}$$

where h is thickness of the shell. Above equations result in

$$N_\alpha - (e_o a)^2 \nabla^2 N_\alpha = \tfrac{K}{R}\left[\frac{\partial u}{\partial \alpha} + \mu_1\left(\frac{\partial v}{\partial \beta} + w\right) - c^2 \frac{\partial^2 w}{\partial \alpha^2}\right] \tag{16}$$

$$N_\beta - (e_o a)^2 \nabla^2 N_\beta = \tfrac{K k_1}{R}\left[\frac{\partial v}{\partial \beta} + \mu_2 \frac{\partial u}{\partial \alpha} + w + c^2\left(\frac{\partial^2 w}{\partial \beta^2} + w\right)\right] \tag{17}$$

$$S_\alpha - (e_o a)^2 \nabla^2 S_\alpha = \tfrac{K k_2}{R}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} - c^2\left(\frac{\partial^2 w}{\partial \alpha \partial \beta} - \frac{\partial v}{\partial \alpha}\right)\right] \tag{18}$$

$$S_\beta - (e_o a)^2 \nabla^2 S_\beta = \tfrac{K k_2}{R}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + c^2\left(\frac{\partial^2 w}{\partial \alpha \partial \beta} + \frac{\partial v}{\partial \alpha}\right)\right] \tag{19}$$

$$M_\alpha - (e_o a)^2 \nabla^2 M_\alpha = -K c^2\left[\frac{\partial u}{\partial \alpha} + \mu_1 \frac{\partial v}{\partial \beta} - \left(\frac{\partial^2 w}{\partial \alpha^2} + \mu_1 \frac{\partial^2 w}{\partial \beta^2}\right)\right] \tag{20}$$

$$M_\beta - (e_o a)^2 \nabla^2 M_\beta = K k_1 c^2\left(\frac{\partial^2 w}{\partial \beta^2} + w + \mu_2 \frac{\partial^2 w}{\partial \alpha^2}\right) \tag{21}$$

$$M_{\alpha\beta} - (e_o a)^2 \nabla^2 M_{\alpha\beta} = 2Kk_2 c^2 \left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha \partial \beta}\right) \quad (22)$$

$$M_{\beta\alpha} - (e_o a)^2 \nabla^2 M_{\beta\alpha} = Kk_2 c^2 \left(\frac{\partial u}{\partial \beta} - \frac{\partial v}{\partial \alpha} + 2\frac{\partial^2 w}{\partial \alpha \partial \beta}\right) \quad (23)$$

$$Q_\alpha - (e_o a)^2 \nabla^2 Q_\alpha = \frac{Kc^2}{R}\begin{bmatrix} \frac{\partial^2 u}{\partial \alpha^2} - k_2 \frac{\partial^2 u}{\partial \beta^2} + (k_2 + \mu_1)\frac{\partial^2 v}{\partial \alpha \partial \beta} - \\ \frac{\partial^3 w}{\partial \alpha^3} - (2k_2 + \mu_1)\frac{\partial^3 w}{\partial \alpha \partial \beta^2} \end{bmatrix} \quad (24)$$

$$Q_\beta - (e_o a)^2 \nabla^2 Q_\beta = \frac{Kk_1 c^2}{R}\begin{bmatrix} 2\frac{k_2}{k_1}\frac{\partial^2 v}{\partial \alpha^2} - \frac{\partial^3 w}{\partial \beta^3} - \\ \frac{\partial w}{\partial \beta} - \left(2\frac{k_2}{k_1} + \mu_2\right)\frac{\partial^3 w}{\partial \alpha^2 \partial \beta} \end{bmatrix} \quad (25)$$

where $K = E_1 h/(1-\mu_1\mu_2)$, $k_1 = E_2/E_1$, $k_2 = G(1-\mu_1\mu_2)/E_1$, $c^2 = h_o^3/(12R^2 h)$.

Using Kelvin model and Eqs. (11) and (12), we get Kelvin-like nonlocal orthotropic elastic shell model.

The obtained model is as follows

$$\left[\frac{\partial^2}{\partial \alpha^2} + k_2(1+c^2)\frac{\partial^2}{\partial \beta^2}\right]u + \left[(\mu_1 + k_2)\frac{\partial^2}{\partial \alpha \partial \beta}\right]v + \left[6 + \frac{\partial}{\partial \alpha} + c^2\left(k_2\frac{\partial^3}{\partial \alpha \partial \beta^2} - \frac{\partial^3}{\partial \alpha^3}\right)\right]w = \frac{\rho h R^2[1-(e_o a)^2\nabla^2]}{K}\frac{\partial^2 u}{\partial t^2} \quad (26)$$

$$\left[(\mu_1 + k_2)\frac{\partial^2}{\partial \alpha \partial \beta}\right]u + \left[k_2(1+3c^2)\frac{\partial^2}{\partial \alpha^2} + k_1\frac{\partial^2}{\partial \beta^2}\right]v + \left[k_1\frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2)\frac{\partial^3}{\partial \alpha^2 \partial \beta}\right]w = \frac{\rho h R^2[1-(e_o a)^2\nabla^2]}{K}\frac{\partial^2 v}{\partial t^2} \quad (27)$$

$$\left[\mu_1\frac{\partial}{\partial \alpha} - c^2\left(\frac{\partial^3}{\partial \alpha^3} - k_2\frac{\partial^3}{\partial \alpha \partial \beta^2}\right)\right]u + \left[k_1\frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2)\frac{\partial^3}{\partial \alpha^2 \partial \beta}\right]v + \left[\left(1 + \frac{1}{c^2}\right)k_1 + \frac{\partial^4}{\partial \alpha^4} + k_1\frac{\partial^4}{\partial \beta^4} + 2k_1\frac{\partial^2}{\partial \beta^2} + (2\mu_1 + 4k_2)\frac{\partial^4}{\partial \alpha^2 \partial \beta^2}\right]c^2 w + \frac{R^2}{K}(1-(e_0 a)^2\nabla^2)\left[Ew + \eta\frac{\partial w}{\partial t}\right] = -\frac{\rho h R^2[1-(e_o a)^2\nabla^2]}{K}\frac{\partial^2 w}{\partial t^2} \quad (28)$$

where $K = \frac{E_1 h}{1-\mu_1\mu_2}$, medium has stiffness E, and the viscosity of the medium is η and the nonlocal parameter is $\Im = (e_o a)^2$. Two kinds of boundary conditions may be assumed while solving such problems. These three conditions are

Clamped-clamped

$$\alpha = \beta = \gamma = \frac{\partial \gamma}{\partial \alpha} = 0, \quad \text{at} \quad \alpha = 0, \alpha = L/R \quad (29)$$

Clamped-free

$$\begin{cases} \alpha = \beta = \gamma = \frac{\partial \gamma}{\partial \alpha} = 0 & \text{at} \quad \alpha = 0 \\ N_{\alpha\alpha} = M_{\alpha\alpha} = N_{\alpha\beta} = M_{\alpha\beta} = 0 & \text{at} \quad \alpha = L/R \end{cases} \quad (30)$$

where L is the length of CNTs.

Using any combination of above three conditions we come close to nonlocal Flugge's shell model. Above system of equations is the nonlocal orthotropic Kelvin-like shell model for CNTs. To understand the waves propagating in CNTs, we need to derive the dispersion relations.

2.2 Application of wave propagation approach

Here, we will discuss wave solutions for single-walled carbon nanotubes. Over the past several years vibration of tube/shell and plate structures of various configurations and boundary conditions have been extensively studied (Hussain *et al.* 2018a, Hussain *et al.* 2018b, Hussain *et al.* 2018c, Hussain and Naeem 2018b, Hussain *et al.* 2019a, Hussain *et al.* 2019b, Hussain *et al.* 2020a, Sehar *et al.* 2020, Hussain *et al.* 2020b).

The solutions of system of Eqs. (26)-(28) for axisymmetric waves is given by Wang and Gao (2016)

$$\begin{cases} u(\alpha, t) = U e^{ik\left(\alpha - \frac{vt}{R}\right)} \\ v(\alpha, t) = V e^{ik\left(\alpha - \frac{vt}{R}\right)} \\ w(\alpha, t) = W e^{ik\left(\alpha - \frac{vt}{R}\right)} \end{cases} \quad (31)$$

where U, V and W are the amplitudes of waves along the direction of x, y and z respectively, the dimensionless wave vector in the longitudinal direction is $k = \frac{\pi m R}{L}$, in longitudinal direction m is the half axial wave number and ν is the wave phase velocity.

Substituting Eq. (31) in system of Eqs. (26)-(28) and simplifying, in matrix form, we get the following system

$$[M^{(1)}(k,\nu)]_{3\times 3}\begin{bmatrix} U \\ V \\ W \end{bmatrix} = [0 \quad 0 \quad 0]^T \quad (32)$$

For the nontrivial solution of above equation, we have

$$Det\left[M^{(1)}(k,\nu)\right] = 0 \quad (33)$$

Table 1 Comparison of Kelvin's model CNT frequencies with Loy *et al.* (1999)

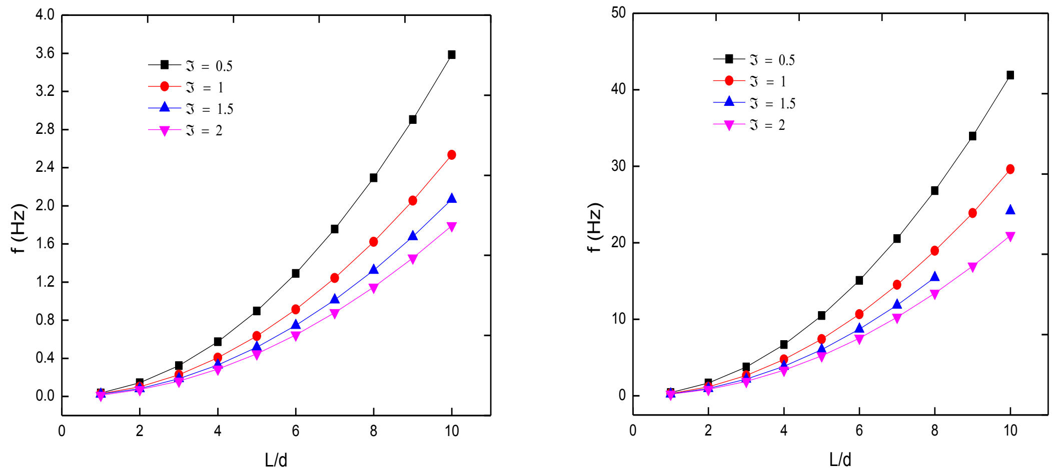
L/R	h/R	Method	N = 1	2	3	4	5	6
20	0.01	Loy *et al.* (1999)	0.016102	0.009382	0.022105	0.042095	0.06801	0.09973
		Present	0.016101	0.009378	0.022103	0.042094	0.04209	0.09973

Table 2 Kelvin's model frequencies of clamped SWCNTs (*h/R*=0.05, *L/R*=2.5)

M	V	N	Heydarpour *et al.* (2014)	Present
0	0.12	7	0.6240	0.6228
		9	0.6240	0.6234
		11	0.6240	0.6239
	0.17	7	0.8157	0.8143
		9	0.8157	0.8152
		11	0.8157	0.8155
	0.28	7	0.8553	0.8541
		9	0.8553	0.8547
		11	0.8553	0.8550

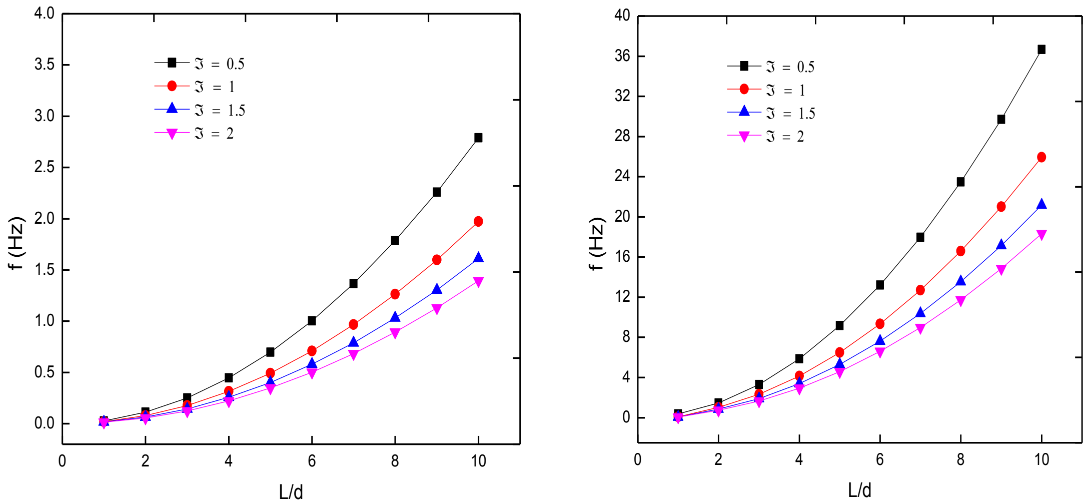
3. Results and discussion

In this portion of writing, the significance of boundary conditions on the vibration behavior of single-walled CNT is investigated employing wave propagation approach. This study specifically scrutinizes the small scale effect in the vibration analysis of single-walled CNT. It is assumed that



(a) (b)

Fig. 4 Aspect ratios against frequencies of C-C zigzag (a) (9, 0), (b) (16, 0) with $\Im$=0.5, 1, 1.5, 2



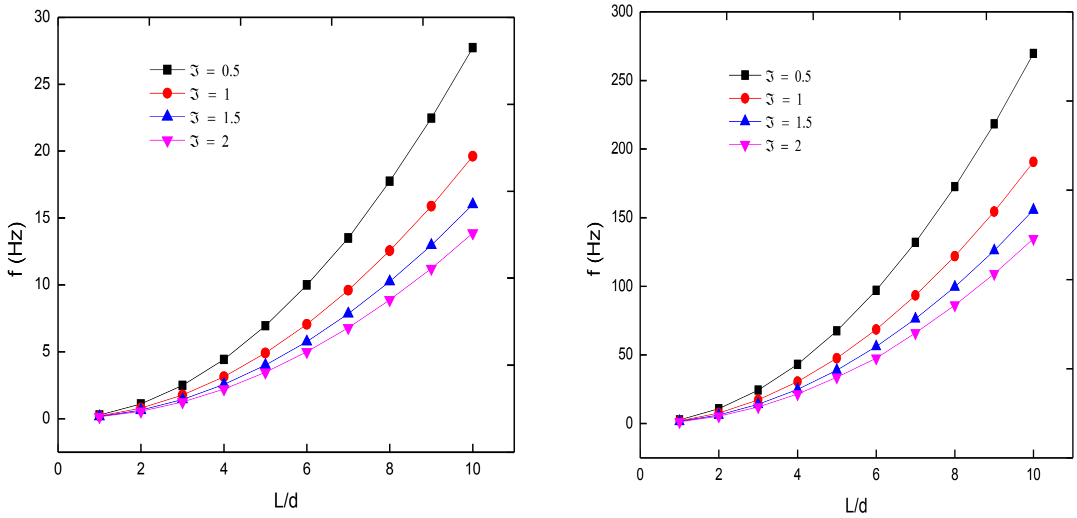
(a) (b)

Fig. 5 Aspect ratios against frequencies of C-F zigzag (a) (9, 0), (b) (16, 0) with $\Im$=0.5, 1, 1.5, 2

the mass density of CNTs is 2300 kg/m^3 (Gupta *et al.* 2010), with the bending rigidity (*EI*) of (5.1122×10^{-9} nm) (Gupta *et al.* 2010, Swain *et al.* 2013). Moreover, distinguished values of inner tube radius together with nonlocal parameter signify the present non-local shell-based model to analyze frequency spectra. CNT is well known structure in shapes of zigzag and chiral, here the orthotropic vibration analysis is carried out subjected to two conditions C-C, C-F. For the convergence rate of CNT, the non-dimensional frequency parameters enumerated in the current work, i.e., using nonlocal Kelvin's model, are happened to be in a good consistency along with the so-called exact results furnished (Loy *et al.* 1999), those were established by working out with the deformation theory provided in Table 1.

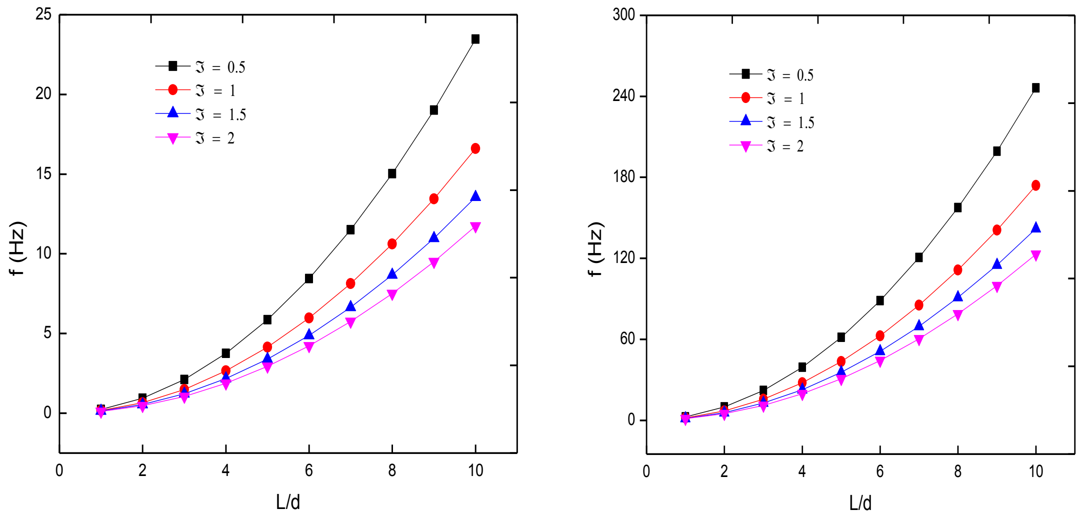
The percentage difference is negligible as *n*=1, 3, 4 are 0.006%, 0.01%, 0.002% and at *n*=2 by 0.0061% and present nonlocal Kelvin's model result are lower than equivalent results executed by Loy *et al.* (1999). The frequency parameters for circumferential wave numbers *n*=5, 6 are same with the outcomes of Loy *et al.* (1999). The results are well matched as shown in Table 2 with the result of Heydarpour *et al.* (2014). The frequency values of zigzag (8, 0) SWCNTs at ($\Im$=0.5, 1, 1.5, 2), when *L/d* (=1~10) are 0.0358, 0.0297, 0.0242, 0.0179 and 3.5842, 2.5344, 2.0613, 1.7921 as shown in the Fig. 4. As it is noted that by increasing the nonlocal parameter, the frequencies decrease. Similarly for zigzag (16, 0), the frequency values are 0.4190, 0.2963, 0.2419, 0.2095 and 41.9000, 29.6278, 24.1910, 20.9500 as shown in the Fig. 4.

For this case, the frequency curves are much lower than that of above clamped-clamped CNTs. In deepness, to understand the vibration characteristics of carbon nanotubes, namely zigzag carbon nanotubes (9, 0), (16, 0), with bending rigidity, different nonlocal parameters and length-to-diameter ratios of 1~10 are considered and the results are discussed. Fig. 5 show the C-F frequencies of different zigzag indices with different nonlocal parameters. Next, the frequency values with C-F zigzag (9, 0) at (*L/d*, $\Im$)=(1, 0.5 & 2) are *f* (Hz)~0.0279, 0.0140 and at (*L/d*, $\Im$)=(10, 0.5 & 2) are *f* (Hz)~2.7905, 1.3952 as shown in the Fig. 5. For the same parameter with C-F zigzag (16,



(a) (b)

Fig. 6 Aspect ratios against frequencies of C-C chiral (a) (13, 6), (b) (23, 8) frequencies versus aspect ratio with $\Im$=0.5, 1, 1.5, 2



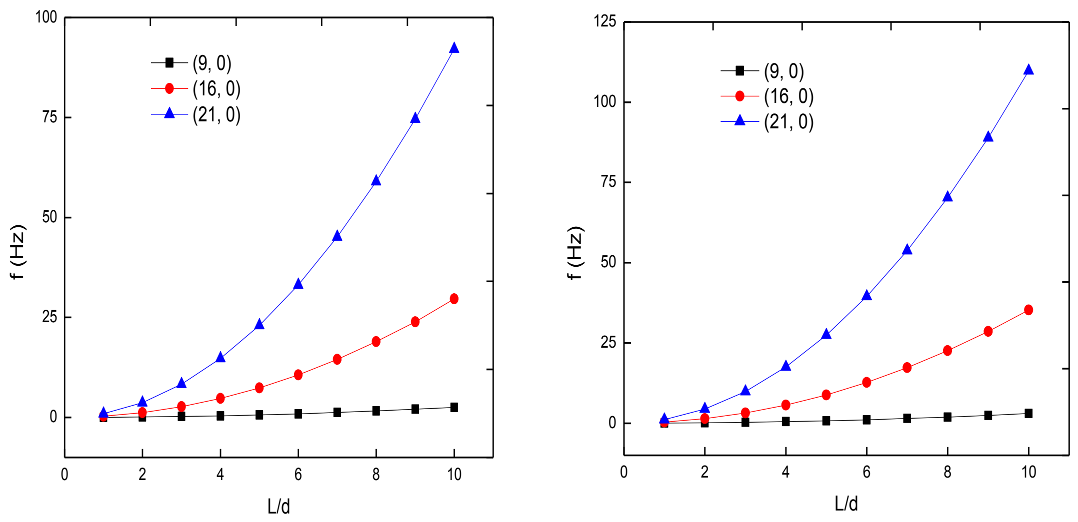
(a) (b)

Fig. 7 Aspect ratios against frequencies of C-F chiral (a) (13, 6), (b) (23, 8) with $\Im$=0.5, 1, 1.5, 2

0), the computed values are *f* (Hz)~36.6680, 18.3340 as shown in Fig. 5. It is again noted that the frequency peaks of C-C zigzag are greater than C-F zigzag with same indices.

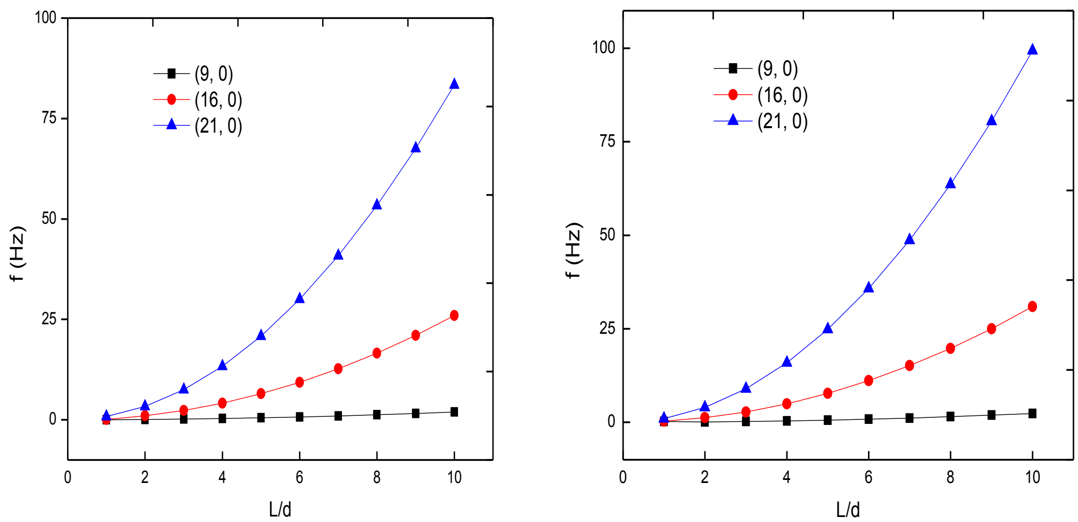
Figs. 4-5 show the variation of the frequency with zigzag indices (9, 0), and (16, 0) with different nonlocal p.

Fig. 6, show that C-C chiral SWCNTs with (13, 6), (23, 8) with different nonlocal parameters. When $\Im$=2 then the frequency peaks are 0.1387, 0.5547, 1.2481, 2.2188, 3.4669, 4.9923, 6.7951, 8.8752, 11.2326, 13.8675 are shown in the Fig. 6. In Fig. 6, for C-C (=23, 8), with same parameters the 1st ten frequencies at $\Im$=0.5 and 2 are 2.6958, 10.7833, 26.2624, 43.1331, 67.3955, 97.0496, 132.0953, 172.5326, 218.3616, 269.5822 and 1.3472, 5.3886, 12.1244, 21.5566, 33.6788, 48.4975, 66.0105, 86.2178, 109.1808, 134.7152. To illustrates the influence of different nonlocal parameter on the natural frequencies for clamped free chiral SWCNTs with indices (13, 6) and (23, 8) based on nonlocal Kelvin's model as shown in Fig. 7. These figures indicated that, obviously, the C-C FNF value is higher than that of C-F value of chiral SWCNTs. As, it is noted that the natural frequency decreases with increasing non-local parameters ($\Im$=0.5, 1, 1.5, 2). It has been observed that from zigzag and chiral SWCNTs case for both C-C and C-F, that the frequency patterns of these nanotubes are clearly visible at peaks. In comparison of frequencies with zigzag and chiral tubes we get a new phenomenon according to the structure of the tube. Due to easy deformation in the cross-section of tube, the frequencies are totally different of zigzag and chiral tubes. Sometimes, when cross section is deformed but not remains circular then different irregular circumferential waveforms, torsional and longitudinal modes can be observed. It is predicted that cross section have no deformation in zigzag and chiral tubes. The frequency values for zigzag tubes with four different nonlocal parameters



(a) (b)

Fig. 8 Aspect ratios against frequencies of C-C (9, 0), (16, 0) and (21, 0) zigzag SWCNTs with (a) EI=5.1122e^{-9} nm (b) EI=7.2617e^{-9} nm and $\mathfrak{T}$=1



(a) (b)

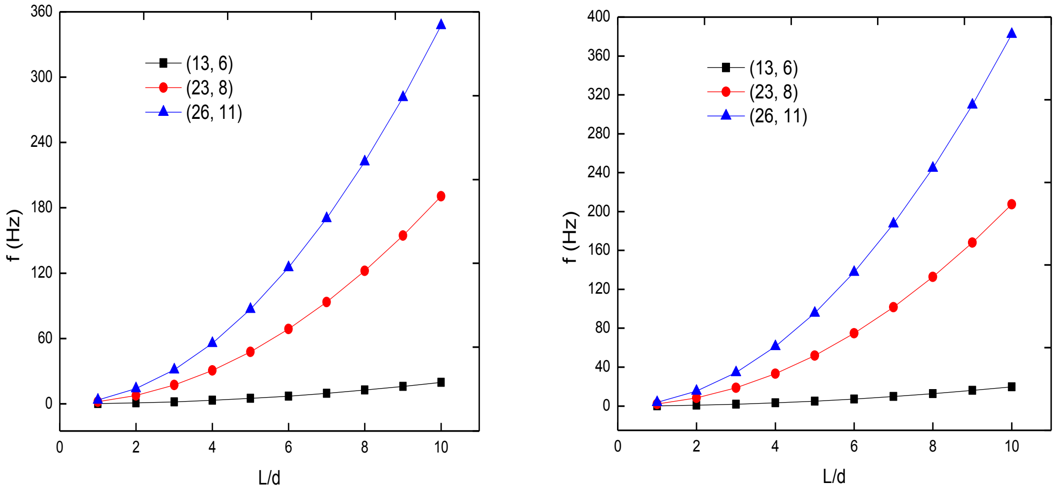
Fig. 9 Aspect ratios against frequencies of C-F (9, 0), (16, 0) and (21, 0) zigzag SWCNTs with (a) EI=5.1122e^{-9} nm (b) EI=7.2617e^{-9} nm and $\mathfrak{T}$=1

are slightly greater than the corresponding values of four nonlocal parameters in chiral case. Therefore, the longitudinal and flexural rigidity of the chiral CNTs are expected to be lower than those of the zigzag CNTs. Figs. 8-9, show the natural frequency behavior of the calculated SWCNT system under bending rigidity (EI). These figures show the frequencies of zigzag (9, 0), (16, 0) and (21, 0) SWCNTs, computed with nonlocal parameter $\mathfrak{T}$=1 based on Kelvin's model. It is evident from these figures that the FNF C-C, C-F=(9, 0), (16, 0) values are lower than C-C, C-F=(21, 0). As indicated by the figures that the fundamental frequencies increases with the increase of aspect ratio and its value increases with the bending rigidity.

Figs. 10-11 show the natural frequency behavior of the calculated SWCNT system under bending rigidity (EI) parameters. A trend of increasing frequencies of indices with bending rigidity is as (26, 11)>(23, 8)>(13, 6).

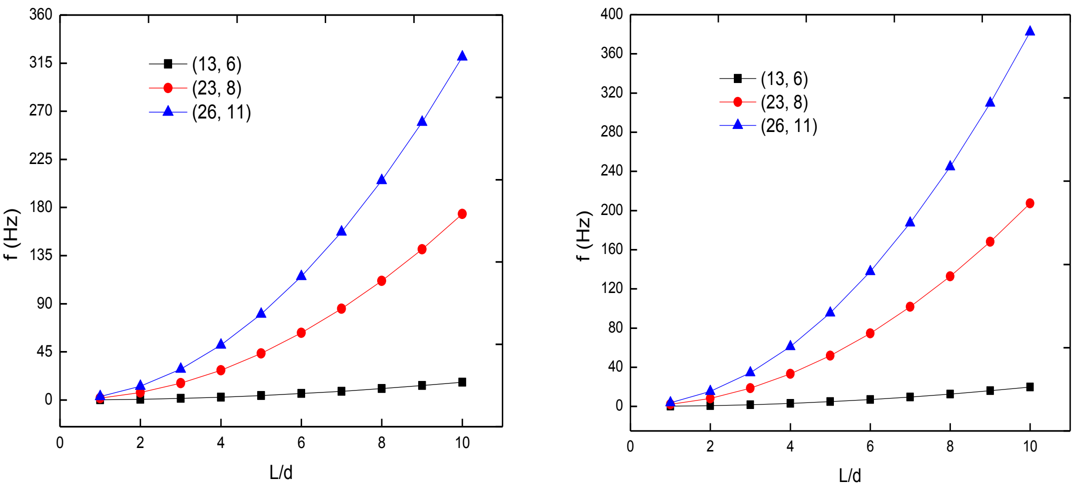
4. Conclusions

A comprehensive estimation regarding nonlocal Kelvin's model based on wave propagation approach has been considered for vibrational behavior of the SWCNTs with distinct nonlocal parameters. Vibrations of SWCNTs for zigzag indices (9, 0), (16, 0), (21, 0) and chiral indices (13, 6), (23, 8), (26, 11) have been analyzed. We developed a new model from the combination of orthotropic nonlocal Kevin's model with wave propagation approach. It is noted that the frequencies of C-C is higher than that of C-F. Also, Kelvin's theory has been utilized for first time to consider the effects of bending rigidity on SWCNTs vibration. This modified model has less complication and has been compared with the earlier methods. The computational results indicated that there is inverse relation of nonlocal parameters and frequencies. The frequency curves of



(a) (b)

Fig. 10 Aspect ratios against frequencies of C-C (13, 6), (23, 8) and (26, 11) chiral SWCNTs with (a) $EI=5.1122e^{-9}$ nm (b) $EI=7.2617e^{-9}$ nm and $\mathfrak{T}=1$



(a) (b)

Fig. 11 Aspect ratios against frequencies of C-F (13, 6), (23, 8) and (26, 11) chiral SWCNTs with (a) $EI=5.1122e^{-9}$ nm (b) $EI=7.2617e^{-9}$ nm and $\mathfrak{T}=1$

clamped-free are lower throughout the computation than the clamped-clamped carbon nanotube. The obtained results show that by increasing aspect ratio of carbon nanotubes, frequency value increases at all boundary conditions. In our measurement we indicated that with higher aspect ratio, the boundary conditions have a momentous influence on vibration of CNT. It can be concluded that frequencies would increase by increasing of the bending rigidity. This means that smaller effects play an important role in predicting SWCNT frequencies, which local theory cannot capture. For future concerns, the present model can be used for viscoelastic vibration of CNTs.

Declaration of conflicting interests

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