Evaluation of moment amplification factors for RCMRFs designed based on Iranian national building code

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Abstract. Geometric nonlinearity can significantly affect load-carrying capacity of slender columns. Dependence of structural stability on columns necessitates the consideration of second-order effects in the design process of columns, appropriately. On the whole, the design codes present a simplified procedure for second order analysis of slender columns. In this approximate method, the end moments of columns resulted from linear analysis (first-order) are multiplied by the recommended moment amplification factors of codes to achieve magnified moments of the second-order analysis. In the other approach, the equilibrium equations are directly solved for the deformed configuration of structure, so the resulting moments and deflections contain the influence of slenderness and increase more rapidly than do loads. The aim of this study is to evaluate the accuracy of moment amplification factors of Iranian national building code whose provisions are similar to the ACI requirement. Herein, finite element method is used to achieve magnified end moments of reinforced concrete moment resisting frames, and the outcomes are compared with the moments acquired based on the proposed approximate method by Iranian national building code. The results show that the approximate method of Iranian code for calculating magnified moments has significant errors for both unbraced and braced columns.

Keywords: geometric nonlinearity; slender columns; design process; linear analysis; reinforced concrete

1. Introduction

When the equations of equilibrium are achieved for initial configuration of the structure while neglecting the deflections effect on internal forces of elements, the analysis is renowned as a first-order. Contrary to this analysis, in a second-order analysis, the equations of equilibrium are derived based on deformed configuration of elements. Bending moments in the former, are called firstorder moments and the additional moments in the latter, are called second-order moments. For slender columns under axial load, the lateral deflection increasing continuously, amplifies the second-order moment (P- δ moment), and the variation of the nodal displacement of the structure generates a different type of second-order moment (P- Δ moment) (Fong et al. 2010). Although, these moments cannot be contained in the first-order linear analysis, due to the simplicity of this analysis and this fact that many engineering computations and computer programs are based on first-order analyses, the design codes have proposed a simplified methods to alter the outcomes of a first-order

*Corresponding author, Assistant Professor E-mail: m.eazadpanah@yahoo.com analysis to consider the second-order effects, indirectly. It is worth emphasizing that against to these simplified methods, it is possible that material and geometric nonlinearities encompassing cracking of the concrete, yielding of the reinforcement, tracing structural elements motion from the initial to the last configuration and so on, are taken into consideration, directly. In spite of the accuracy of nonlinear analyses, these kinds of analyses are so complex, tedious and time-consuming which is against to the designing proposes. Over the last three decades, material and geometric nonlinearities have been addressed in many studies (Habibi and Bidmeshki 2019, Izadpanah and Habibi 2018, Habibi and Bidmeshki 2018, Vetr *et al.* 2017, Oveisi *et al.* 2017, Wan and Zha 2016 and Thombare *et al.* 2016).

Majority of national codes such as, ACI-318 (2014), Eurocode 2, Iranian national building code (9th issue) (INBC9), present the moment magnifier procedure to consider the second order effect to design slender RC columns. For second order analysis of slender RC columns, Eurocode 2 proposed two simplified procedure: one based on nominal stiffness and another based on nominal curvature. De Araujo (2017) showed that the achieved outcomes of these two procedures have large differences. They compared the results of these two methods with experimental outcomes of previous studies and concluded that nominal curvature presents more reliable results. Barros et al. (2010) evaluated the nominal curvature method of Eurocode 2 and extended a new definition of the maximum curvature of the column and the deflected shape. They compared their proposed method with Eurocode 2 and

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showed that the extended method presents less steel area than in Eurocode 2.

Du *et al.* (2017) developed a new flexibility-based beam-column element with member imperfection for second-order analysis of steel structures. Areiza-Hurtado and Aristizábal-Ochoa (2019) carried out the second-order analysis of a beam-column on elastic foundation. They considered initial transverse deflections and semi-rigid end-connections subject to transverse load. Applying Fourier series, any type of loads and initial imperfections can be simulated by this method. The abovementioned method can reflect the phenomenon of snap-through, snap-back and the post-buckling behavior.

Bonet et al. (2011) studied effective flexural stiffness of slender RC columns subjected to axial forces and biaxial bending. They derived an equation to achieve effective flexural stiffness of slender RC columns with any shape of cross-section, subjected to combination of axial loads and biaxial bending, both for short-time and sustained loads, normal and high strength concretes. They compared the outcomes of the proposed method with 613 experimental tests from the literature and showed that the results of the proposed method are in good compliance with those of experimental tests. Leite et al. (2013) performed an experimental research on 32 columns tested with unequal eccentricities at the ends. They studied concrete strength, the slenderness, the amount of longitudinal reinforcement, the ratio between eccentricities at the ends and the relative eccentricity applied. Moreover, they compared the experimental results with simplified approaches proposed by Eurocode 2 and by ACI-318 (2008). The outcomes showed that simplified methods introduced in the Eurocode 2 and the ACI-318 (2008) presents more accuracy for normal strength concrete columns than high strength concrete columns. Fong et al. (2010) proposed an advanced analysis design method for composite columns. They evaluated the validity of their presented method via comparing its outcomes with those of Eurocode 4 and some laboratory tests. They illustrated that their method is a general and conventional method that can be used to design composite structures. Hellesland (2009) presented a modified approximate story magnifier approach to consider second order effects via two separate flexibility factors. He showed that the proposed approach is practical and viable. Zubydan (2010) studied inelastic behavior of steel frames and extended a simplified model including the residual stresses to analysis of planer frames. The updated Lagrange coordinates and the Newton Raphson method are employed in this extended procedure. It was illustrated that this method presents acceptable results and faster rate of convergence. Iu (2015) presented generalized element load method with various load effects. A remarkable feature of this element is to derive the continuous first- and secondorder elastic displacement and force solutions along an element without loss of accuracy. An optimal design of frame structures based on ordinary first-order analysis was performed by Gil-Martín et al. (2006). Karaca and Turkeli (2014) studied the influence of slenderness on wind response of industrial reinforced concrete chimneys. They presented some graphs representing the influence of slenderness on chimneys. They also proved that slenderness can affect the wind responses of slender industrial reinforced concrete chimneys, significantly.

Studying the literature shows that the evaluation of the accuracy of simplified method of Iranian national building code to alter the moments of the first-order analysis accounting for the second-order moments is relatively rare. In present study, firstly some Reinforced Concrete Moment Resisting Frames (RCMRFs) with wide variety of bays and stories are designed. After that, once, the first-order analysis of each designed frame is performed and the second-order moments of slender columns of each frame are achieved through simplified procedure of INBC9. Then, the finite element method is applied to acquire the magnified end moments of these two kinds of analyses are compared.

2. Second order effects in slender columns

The general procedure to predict the actual behavior of slender columns is the nonlinear analysis in which the material nonlinearity coupled with geometric nonlinearity are simultaneously considered. In the material nonlinearity, cracking of concrete, yielding of reinforcements, confining effect and so on should be appropriately considered. In the geometric nonlinearity, the large deformation and strain are considered using the Lagrangian description. To solve nonlinear problems, the well-known incremental-iterative procedures are widely used. Considering simultaneously both kinds of nonlinearities requires intensive computations that is beyond the design purposes. Therefore, design codes have proposed some simplified methods to approximate the slenderness effects.

In the simplified method of INBC9 whose provisions are similar to the ACI-318 (2014) requirement, moments acquired applying a first-order frame analysis which does not consider slenderness effects are multiplied by a moment magnifier accounting for the slenderness effects. It is worth emphasizing that in the linear elastic analysis, the cracking effect should be considered. To do so, the moment of inertia of columns and beams are obtained according to INBC9, as follows unless a more rigorous analysis is used (for columns, Eq. (1) and for beams Eq. (2))

$$I_{c}=0.7 I_{g}$$
 (1)

$$I_b = 0.35 I_g$$
 (2)

where I_C and I_b are the cracked moment of inertia of column and beam, respectively. I_g is the moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.

In Iranian building code, non-sway and sway frames are considered, separately. A non-sway story is a story for which the lateral displacements are negligible compared with the first-order moments due to lateral loads. According to Iranian building code, the stability index (Eq. (3)) for a non-sway story does not exceed 0.05.

$$Q = \frac{\sum N_u \delta_u}{H_u h_s} \tag{3}$$

In the above relation, ΣN_u is the total factored vertical load and H_u is the horizontal story shear, in the story being assessed. δ_u is the first-order relative lateral deflection between the top and the bottom of the considered story due to H_u . h_s is the height of considered story. According to Iranian building code, for a not braced and braced column against side sway, slenderness effects can be neglected if Eqs. (4)-(5) are satisfied.

$$\frac{Kl_u}{r} \le 22 \tag{4}$$

$$\frac{Kl_u}{r} \le \min\left(34 - 12\frac{M_1}{M_2}, 40\right) \tag{5}$$

where K is effective length factor, l_u is unsupported length of column, and r is radius of gyration of cross section (to simplify, 0.30 times the dimension in the direction stability is being calculated for rectangular columns). M_1 and M_2 are lesser and greater factored end moments. If the factored column moments are small or zero, the design of slender columns should be based on the minimum eccentricity provided in Eq. (6). On the other hand, M_2 includes the effects of imperfections and should be more than $M_{2,\min}$ (Eq. (7)).

$$e_{\min} = 15 + 0.3h$$
 (6)

$$M_{2,\min} = P_u(15 + 0.3h)$$
 (7)

where P_u is factored axial force, $M_{2,\min}$ is minimum value of M_2 and h is the overall thickness of the member perpendicular to the axis of bending. The sign convention for $\frac{M_1}{M_2}$ is positive if bent in single curvature and negative if bent in double curvature. The factored moment (M_c) applied for design of columns, in a non-sway frame is achieved using Eq. (8).

$$M_c = \delta M_2$$
 (8)

where δ is the magnification factor calculated from Eq. (9).

$$\delta = \frac{c_m}{(1 - N_u / 1.15 \phi_c N_c)} \ge 1$$
(9)

where N_u and N_c are the factored axial force and critical buckling load (Eq. (10)). C_m can be computed through Eq. (11), when non transverse load applied between supports. Unless C_m equal 1. ϕ_c is the strength reduction factor (0.65 in this study).

$$N_c = \frac{\pi^2 E I_e}{\left(\kappa I_u^2\right)} \tag{10}$$

$$C_m = 0.6 + 0.4 \frac{M_{1b}}{M_{2b}}$$
 (11)

 M_{1b} and M_{2b} are factored end moments on a column due to loads that cause no appreciable side sway, where, in order to simulate the variation of stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve, the Eq. (12) is proposed. To simplify the Eq. (12), the approximation relation in Eq. (13), can be used as well.

$$EI_{e} = \frac{(0.2E_{c}I_{g} + E_{s}I_{se})}{1 + \beta_{d}}$$
(12)

$$EI_e = 0.25E_c I_g \tag{13}$$

where β_d is the ratio of maximum factored sustained shear within a story to maximum factored shear in that story. I_{se} is moment of inertia of reinforcement about centroidal axis of member cross section. E_c and E_s are modulus of elasticity of concrete and modulus of elasticity of reinforcement, respectively.

The magnified moments in the two ends of an individual column, in a sway frame, are derived using Eqs. (14)-(15).

$$M_1 = M_{1b} + \delta_s M_{1s} \tag{14}$$

$$M_2 = M_{2b} + \delta_s M_{2s} \tag{15}$$

where M_1 and M_2 are the magnified moments in two ends of column, achieved applying a first-order elastic frame analysis. M_{1s} and M_{2s} are the factored end moments on a column due to loads which cause appreciable side sway, acquired via a first-order elastic frame analysis. δ_s is moment magnification factor to capture lateral drift deriving from both lateral and gravity loads. The values of $\delta_s M_{1s}$ or $\delta_s M_{2s}$ can be obtained using one of three different approaches, 1) Second-order elastic analysis, 2) Q method (limited to stability indices which do not exceed 1/3) (Eq. (16)).

$$\delta_s = \frac{1}{1-Q} \ge 1 \tag{16}$$

and 3) using the Eq. (17)

$$\delta_s = \frac{1}{(1 - \sum N_u / 1.15 \phi_c \sum N_c)} \ge 1$$
(17)

where $\sum N_u$ and $\sum N_c$ are the summation of all the factored vertical loads and the summation of critical buckling loads for all sway-resisting columns, in a story, respectively. In this study, due to the limitations of the Q method, magnified moments are achieved using the third above mentioned methods means Eq. (17). Comparing the simplified methods of INBC9 and ACI-318 (2014) to approximate the slenderness effects shows that both codes presented the same regulations to derive the moment magnification factors.

Another method that is used in this study to acquire the magnified moments, is second-order elastic analysis, in which the stiffness matrix of each vertical member is a combination of elastic stiffness and geometry stiffness matrix (Eqs. (18)-(19)) (Choi and Yoo 2009)

$$[K_{E}] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0\\ & \frac{12EI}{L^{3}} & \frac{6EI}{L^{2}} & 0 & -\frac{12EI}{L^{3}} & \frac{6EI}{L^{2}}\\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^{2}} & \frac{2EI}{L}\\ & & & \frac{EA}{L} & 0 & 0\\ & & & & L & 0 & 0\\ Symm. & & & \frac{12EI}{L^{3}} & -\frac{6EI}{L^{2}}\\ & & & & & \frac{4EI}{L} \end{bmatrix}$$
(18)



Fig. 1 Geometry of studied frames



where E is the modulus of elasticity, I and L are the second moment of area, and the length of column, respectively. P is the axial force. In present study, to specify the buckling load, the presented method in Choi and Yoo (2009) is used. These matrices are combined for each member and then the stiffness matrices of structural members are assembled to form the global stiffness matrix of each frame. In the first step, since the axial force of each column to form the geometry matrix is unknown, the stiffness matrix of each vertical member is considered as Eq. 18. The obtained axial forces in the first step are applied to form the geometry matrix in the next step. After acquiring the total stiffness matrix of structure, the second-order analysis is performed and nodal displacements are acquired. Then, the axial force of each member based on the obtained nodal displacements is obtained and the stiffness matrix is updated. The secondorder analysis is carried out via new stiffness matrix and axial forces and nodal displacements are acquired. This process is repeated until the differences between nodal displacements and axial forces with previous ones satisfy the convergence criterion.

3. Numerical study

In the current study, eight RCMRFs with various stories (S) and bays (B) have been considered (Fig. 1). The numbers of stories have been assumed to be one, three, five, seven, nine, eleven, thirteen and fifteen. The beam and column element numbers for a general frame with n bays and m stories are presented in Fig. 2. These frames are designed based on the criteria of Iranian national building



Table 1 The considered load combinations

Load combination number	Load coefficients
1	1.25DL + 1.5LL
2	DL + 1.2LL + 0.84E
3	0.85DL + 0.84E

code and Iranian seismic code. In the aforementioned frames, the height of each story is 3.2 m and the length of each bay is 4 m. All the frames lie on a soil type B (rock site). The frames are designed for high risk seismic hazard. The distributed gravity dead and live loads imposed on the beams are assumed to be 22 and 8 KN/m. The concrete is assumed to have cylinder strength of 30 MPa, strains of 0.002 and 0.0035 at maximum strength and ultimate strength, respectively. The concrete has a modulus of elasticity of 26621 MPa. The steel has a yield strength of 400 MPa and a modulus of elasticity of 200,000 MPa. The sectional properties of the designed beams and columns have been detailed by Rohani (2016).

To obtain the magnified moments, three different load combinations have been considered (Table 1).

In Table 1, DL and LL are dead and live loads, respectively. E presents the seismic load (lateral load). In Iranian national code, a same magnification factor is presented for whole the column, whereas in the second order method, a unique magnification factor is achieved for each end of a column; therefore, in this study, the peak of magnification factors of two ends for each column is compared with the proposed magnification factor of INBC9. For each load combination, comparing the above mentioned magnification factors of the designed frames are presented in the following sections.

In the first load combination, simply the gravity loads are considered. For this load combination, in lacking of lateral loads, the slenderness effect for the designed frames (frames are symmetric) is limited to lateral deformations along individual members (P- δ) and side sway of the overall structure is negligible (P- Δ). For this load combination, all columns fall into the non-sway story. In Fig. 3, for each frame, the moment magnification factors of nonlinear analysis and INBC9 are compared. It should be pointed out that for some columns; the value of moment has been close to zero and the magnification factor has not been acquired for them.



Fig. 3 The moment magnification factors of nonlinear analysis and Iranian building code for frame: (a) S1B1 (b) S3B2 (c) S5B2 (d) S7B3 (e) S9B3 (f) S11B4 (g) S13B4 (h) S15B4, subjected to load combination 1

In Fig. 3, the accuracy of proposed relation of INBC9 for considering the effect of lateral deformations along individual members (P- δ) on the column end moments is assessed. The outcomes of nonlinear analysis show that for some columns (frame S7B3: columns 10 and 11, frame S9B3: columns 34 and 35, frame S13B4: columns 2,4,52 and 54 and frame S15B4: columns 2 and 4), the P- δ effect causes a reduction in end moments. For all columns of frames S1B1, S3B2, S5B2, S11B4 and S15B4, the maximum difference between nonlinear analysis and INBC9 is lower than 1 percent. In frame S7B3, the error of

the proposed relations of INBC9 for columns 2, 3, 6 and 7 are around 7 percent. In frame S9B3, the maximum gaps between results of nonlinear analysis and INBC9 are related to columns 2 and 3 and are around 7.4 percent. In frame S13B4, for columns 2 and 4, the error of the relation proposed by Iranian building code is lower than 2 percent. It can be concluded that despite of conservative performance of INBC9 for majority of columns in nonsway story, for some columns, the outcomes of simplified method of INBC9 is unreliable.

In the second load combination, the gravity loads



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Fig. 4 The moment magnification factors of nonlinear analysis and Iranian building code for frame: (a) S1B1 (b) S3B2 (c) S5B2 (d) S7B3 (e) S9B3 (f) S11B4 (g) S13B4 (h) S15B4, subjected to load combination 2

coupled with lateral load are considered, simultaneously. For this load combination, the slenderness effect includes both lateral deformations along individual members (*P*- δ) and side sway of the overall structure $(P-\Delta)$. In Fig. 4, for each frame, the outcomes of nonlinear analysis and the proposed moment magnification factors of INBC9 are presented.

Both columns for frame S1b1 fall into braced columns

and the moment magnification factors for both columns are compliant as shown in Fig. 4(a). For frame S3b2, all columns apart from 4, 5 and 6 are braced columns. As manifested in Fig. 4(b), for unbraced columns, the outcomes of nonlinear analysis and simplified method of INBC9 are in agreement but for the most of braced columns, the proposed moment magnification factors of INBC9 are unreliable. For frame S3b2, the maximum error

51 53 59

Nonlinear Analysis

Column number

Column number

(d)

23 25 27 29 31 33 Column number

(f)

Nonlinear Analysis

Nonlinear Analysis
 Iranian Code

■Nonlinear Analysis

Iranian Code

Iranian Code

(b)

Iranian Code



(g)

Fig. 5 The moment magnification factors of nonlinear analysis and Iranian building code for frame: (a) S1B1 (b) S3B2 (c) S5B2 (d) S7B3 (e) S9B3 (f) S11B4 (g) S13B4 (h) S15B4, subjected to load combination 3

(20 percent) accounts for the first column. In Fig. 4(c), the columns of the first four stories fall into non-sway story and those of fifth story are unbraced. Although INBC9 presents unreliable results for unbraced columns, the maximum error is lower than 5 percent. For frame S7B3, the first and last stories are sway stories. For the most of columns of this frame, the moment magnification factors of INBC9 are underestimated. The error percentage of columns 1, 6, 7, 10, 11, 14 and 15 are close to 10. For frame S9B3, the second to sixth stories are sway stories. For the majority of

columns of this frame, the moment magnification factors of INBC9 are unreliable and the maximum error (around 12%) belongs to columns 6, 7, 10, 11, 14 and 15. As shown in Fig. 4(e), the value of the proposed moment magnification factor of INBC9 for column 46 is about 27 percent lower than that of nonlinear analysis. The main reason for this gap refers to the shortcoming of simplified method of INBC9 in assuming one moment magnification factor for both ends of columns. For example, in nonlinear analysis, for column 46, the obtained moment magnification factor for top end (with

Column number

greater moment) is 1.03889 and for bottom end (with lower moment) is around 1.3646, whereas the achieved moment magnification factor of Iranian code is 1 for this column. For columns 1 to 5, the obtained moment magnification factors of nonlinear analysis are around 10 percent more than those of INBC9. For frame S13B4, apart from the columns of 12th and 13th, all vertical members are unbraced. For this frame, it is obviously observed that for overwhelming majority of the columns, the obtained magnifier factors by INBC9 are underestimated. The situation of S15B4 is similar to S13B4. The maximum errors of 11 and 24 percent have been achieved for columns 4 and 12 of frames S13B4 and S15B4, respectively.

In the last but not least load combination, the dead and earthquake loads while neglecting live load are chosen. This load combination is defined for overturning control of frames. For this load combination, although both *P*- δ and *P*- Δ can affect the end moments of columns, it is predicted that the influence of *P*- δ will be negligible due to the low coefficient of gravity load. In frames S1B1, S3B2, S5B2 and S7B2, all stories are non-sway stories. In frame S9B3, the second and third stories, in frame S11B4, the third to fifth stories, in frame S13B4, the second to fifth stories, and in frame S15B5, the second to eighth stories are non-sway. In Fig. 5, the obtained moment magnification factors through nonlinear analysis and INBC9 relations are compared.

Comparing the results shows that for frames S1B1, S3B2 and S5B2, the maximum difference is around 13 percent. For all columns of frames S7B3, S9B3, S11B4 and S13B4, the moment magnification factors of INBC9 are lower than nonlinear analysis. In frame S7B3, the highest error (45 percent) is related to the column 1. The maximum errors of frames S9B3, S11B4 and S13B4 are 11% (column 33), 14% (column 6) and 12% (column 1), respectively. In frame S15B4, the values of moment magnification factors of INBC9, for some columns are overestimated (the third to sixth stories) and for others are underestimated. The maximum error of this frame is around 17 percent and belongs to column 4.

4. Conclusions

In this study, the correctness of the simplified method of Iranian national building code to achieve the amplified firstorder moment for consideration of the second-order effects is evaluated. To do so, end moments of all columns of the studied frames are acquired once through performing the nonlinear analysis directly and again via the first-order moment magnifier factors of Iranian national building code. For the frames, three various load combinations to reflect different types of slenderness effect of columns are assumed. The following outcomes are highlighted:

• All columns subjected to the gravity load combination (the first load combination) fall into the non-sway story. For this load combination, with raising the number of stories, the gap between the outcomes of second-order analysis and simplified procedure of INBC9 increases. Although for all the columns, the maximum difference is a lower than 7.4 percent, it is shown that INBC9 presents lower moment magnification factors for majority of columns.

• For the load combination containing both gravity and lateral loads (the second load combination), both $P-\delta$ and $P-\Delta$ are effective. Some columns are braced and others are unbraced based on the INBC9 criteria. In this load combination, for both low-rise and high-rise frames, the error of simplified method of INBC9 is more than 20 percent. It is proved that the INBC9 moment magnification factors for many of columns are underestimated. The highest error belongs to the nine-story frame.

• For the overturning load combination (the last load combination), $P-\delta$ coupled with $P-\Delta$ can affect the end moments of columns but sure enough, the influence of P- δ due to the low coefficient of gravity load is negligible. For this load combination, apart from some limited columns, INBC9 presents underestimated moment magnification factors and the highest error is around 45 percent that is a considerable amount.

Eventually, it can be concluded that despite of some advantages of the proposed method of Iranian code such as simplicity, straightforwardness and low computational effort, this method has some defects and presents the unreliable outcomes for many cases.

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