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Nominal flexural strength of high-strength concrete beams

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Abstract. The conventional ACI rectangular stress block is developed on the basis of normal-strength concrete column tests and it is still being used for the design of high-strength concrete members. Many research papers found in the literature indicate that the nominal strength of high-strength concrete members appears to be over-predicted by the ACI rectangular stress block. This is especially true for HSC columns. The general shape of the stress-strain curve of high-strength concrete becomes more likely as a triangle. A triangular stress block is, therefore, introduced in this paper. The proposed stress block is verified using a database which consists of 52 tested singly reinforced high-strength concrete beams having concrete strength above 55 MPa (8,000 psi). In addition, the proposed model is compared with models of various design codes and proposals of researchers found in the literature. The nominal flexural strengths computed using the proposed stress block are in a good agreement with the tested data as well as with that obtained from design codes models and proposals of researchers.

Keywords: beams; flexural strength; high-strength concrete; triangular stress block

1. Introduction

The use of high-strength concrete (HSC), f_c'>55 MPa (8,000 psi), has become the most widely used and most consumable building material in the world in recent years. HSC offers reduction in section size, span length, and weight of concrete structural elements when used in high-rise buildings and bridges. In most design codes, the traditional stress block that is developed for normal-strength concrete (NSC) is still being used for the design of HSC elements. This gives a strong motivation to examine the current ACI 318 (2014) provisions for nominal strength calculations for HSC members because they are developed based on NSC columns tests (Bae and Bayrak 2013, ACI 441.1R 2018). Several stress block alternatives to calculate the strength for high-strength concrete members have been proposed; i.e., CEB-FIP Model Code (2010), Mertol *et al*. (2008), NZS 3101 (2006), CSA A23.3 (2004), EN 1992 (2004), Bae and Bayrak (2003), Ozbakkaloglu and Saatcioglu (2004), Ibrahim and MacGregor (1997), Azizinamini *et al*. (1994).

Recently, some researchers have proposed stress block models based on a tested data of HSC beams and columns. Khadiranaikar and Awati (2012) have developed stress-block parameters for wide range of concrete strength. The experimental program includes testing of plain concrete columns, reinforced concrete members such as eccentrically loaded columns, and beams in pure flexure. A generalized equivalent stress block model that works for both light and normal weight HSC is proposed by Yang *et al*. (2013). The coefficients used in the proposed stress block were formulated based on a nonlinear regression analysis through an extensive database of test data.

Designing of HSC members requires a stress block model that best represents the concrete stress-strain characteristics. In this research, the stress block model is determined from the shape of the stress-strain curve of HSC. For the stress-strain relationship of HSC, as the concrete strength increases, the strain increases and reaches a peak value of 0.003. The shape of the ascending part of the relationship becomes more linear and steeper. Similarly, the slope of the descending branch becomes steeper. The general shape of the stress-strain relationship for HSC is similar to a triangle. Hence, a triangular stress distribution is better suited for HSC (Wahidi, 1995). Wahidi (1995) used the experimental results of nine HSC columns tests to compare the triangular stress block and other stress blocks with a proposed modified rectangular stress block. The triangular stress block was slightly more conservative than the modified rectangular stress block. Extending this concept, a triangular stress block is suggested in this paper to calculate the nominal flexural strength of HSC beams possessing a concrete strength above 55 MPa (8,000 psi). The results obtained by using the triangular stress block is compared with the results by using stress blocks of various codes and proposals of researchers. The comparison is done by using test results of 52 tested singly reinforced high-strength concrete beams having concrete strength above 55 MPa (8,000 psi).

2. Research significance

The current ACI 318-14 provisions use a rectangular stress block for all concrete strength. The shape of the stress-strain curve is adjusted by the factor β_1 to account for the higher strength. Above 55 MPa (8,000 psi) there is no further changes in this value. In addition, some design

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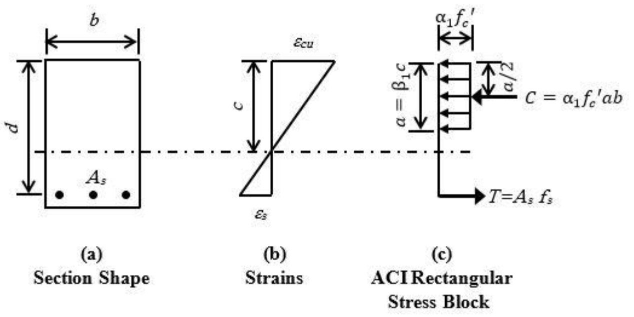

Fig. 1 Rectangular stress block

codes and individuals have proposed alternative stress block models for calculating strength of HSC members. These cases mean that there is no universal agreement on a rational stress block model for high-strength concrete nor is there agreement on when a transition in the stress block parameters should occur. Therefore, this paper proposes that the solution for this problem is to use the ACI rectangular stress block for concrete strength up to 55 MPa (8,000 psi) and replace the ACI rectangular stress block by the triangular stress block proposed in this paper for concrete strength above 55 MPa (8,000 psi).

3. Stress block parameters

3.1 ACI 318 (2014) provisions

The stress block parameters used in the current code provisions were originally proposed by Whitney (1937) and confirmed based on the test results reported by Mattock *et al*. (1961). The tests done by Mattock *et al*. (1961) were for NSC columns having compressive strength less than 55 MPa (8,000 psi). Fig. 1 shows the rectangular stress block parameters of ACI 318 (2014). The intensity of the rectangular block is α_1 *fc'*. The parameter α_1 is constant for all concrete strengths with a value equal to 0.85. The depth of the stress block is $\beta_1 c$, where *c* is the neutral axis depth. The parameter β_1 is equal to 0.85 for concrete strength up to 30 MPa (4,350 psi) and decreases linearly at a rate of 0.08 for each 10 MPa (1450 psi) and should not be less than 0.65. The ultimate compressive strain ε_{cu} at the extreme compression fiber is set to 0.003 for all concrete strengths. It is worth mentioning that an upper limit for the concrete strengths is not specified by the current ACI provisions.

3.2 Mertol et al. (2008)

Mertol *et al*. (2008) derived a stress block parameter based on testing of 21 plain concrete specimens with concrete strengths up to 124 MPa (18,000 psi) and a collected data from other researchers. The collected data were from test results obtained by Hognestad *et al*. (1955), Sargin *et al*. (1971), Nedderman (1973), Kaar *et al*. (1978), Swartz *et al*. (1985), Pastor (1986), Schade (1992), Ibrahim (1994), and Tan and Nguyen (2005). According to Mertol *et al*. (2008), the tested data and the collected data indicated that for HSC a lower bound for the parameters α_1 and β_1 can be 0.75 and 0.65, respectively.

The stress block parameter α_1 proposed by Mertol *et al*. (2008) is set at a value of 0.85 for concrete strength of 69 MPa (10,000 psi) and lower. However, this parameter decreases linearly to reach a minimum value of 0.75 for concrete strength of 103 MPa (15,000 psi) or above. The parameter β_1 has a maximum value of 0.85 for concrete strength less than or equal 28 MPa (4,000 psi) and decreases linearly to a minimum value of 0.65 for concrete strength of 55 MPa (8,000 psi) or above. An ultimate compressive strain of 0.003 is considered applicable for the design purposes. It is interesting to note that the parameter β_1 adopted by Mertol *et al*. (2008) is identical to that of ACI 318 provisions; however, the lower limit of α_1 is reduced linearly from 0.85 to 0.75 and not included in the ACI provisions.

3.3 NZS 3101 (2006)

The New Zealand standards (NZS 3101, 2006) developed stress block parameters based on the test results obtained by Li *et al.* (1995). For concrete strength below 55 MPa (8,000 psi), the α_1 parameter is taken as 0.85. For higher concrete strength, α_1 parameter is reduced linearly to a minimum value of 0.75 at concrete strength of 80 MPa (11,600 psi) and remain constant for concrete strength above 80 MPa (11,600 psi). On the other hand, the β_1 parameter is set at a value of 0.85 for concrete strength up to 30 MPa (4,350 psi) and decreased linearly to a minimum value of 0.65 at concrete strength of 55 MPa (8,000 psi) and above. For the design purposes, the NZS 3101 (2006) standard sets the maximum useable compressive strain at 0.003. The lower limits for α_1 and β_1 parameters are identical to that of Mertol *et al*. (2008).

3.4 CSA A23.3 (2004)

The Canadian standards association (CSA A23.3, 2004) proposed stress block parameters different than the current ACI provisions. Both α_1 and β_1 parameters depend on the concrete compressive strength and vary linearly as the compressive strength increases. The α_1 parameter has a maximum value of 0.85 and decreases linearly at a rate of 0.015 for each 10 MPa (1,450 psi). However, the β_1 parameter has a maximum value of 0.97 and decreases linearly at a rate of 0.025 for each 10 MPa (1,450 psi). Both α_1 and β_1 parameters should not be less than 0.67. According to CSA A23.3 (2004) standards, the maximum strain at the extreme concrete compression fiber shall be assumed to be 0.0035.

3.5 Bae and Bayrak (2003)

Bae and Bayrak (2003) modeled the stress-strain response of unconfined HSC using the suggestion of Popovics (1973) which was then modified by Thorenfeldt *et al*. (1987) and Collins *et al*. (1993). In deriving the stress block parameters, the maximum reliable strain was assumed as 0.0025 because of the issue of cover spalling for HSC columns. The parameter α_1 is equal to 0.85 for concrete strength up to 70 MPa (10,150 psi) and decreases linearly at

Table 1 Rectangular stress block expressions: codes and proposals

Code or proposal	fc' in MPa	fc' in psi	ε_{cu}
ACI 318 (2014)	α_1=0.85 β_1=0.85-0.008(f_c'-30) $0.85 \geq \beta_1 \geq 0.65$	α_1=0.85 β_1=0.85-0.05/1,000X(f_c'-4,000) $0.85 \geq \beta_1 \geq 0.65$	0.003
Mertol *et al*. (2008)	α_1=0.85-0.0029(f_c'-69) $0.85 \geq \alpha_1 \geq 0.75$ β_1=0.85-0.007252(f_c'-28) $0.85 \geq \beta_1 \geq 0.65$	α_1=0.85-0.02/1,000X(f_c'-10,000) $0.85 \geq \alpha_1 \geq 0.75$ β_1=0.85-0.05/1,000X(f_c'-4,000) $0.85 \geq \beta_1 \geq 0.65$	0.003
NZS 3101 (2006)	α_1=0.85-0.004(f_c'-55) $0.85 \geq \alpha_1 \geq 0.75$ β_1=0.85-0.008(f_c'-30) $0.85 \geq \beta_1 \geq 0.65$	α_1=0.85-0.028/1,000X(f_c'-8,000) $0.85 \geq \alpha_1 \geq 0.75$ β_1=0.85-0.05/1,000X(f_c'-4,000) $0.85 \geq \beta_1 \geq 0.65$	0.003
CSA A23.3 (2004)	α_1=0.85-0.0015f_c' $\geq$ 0.67 β_1=0.97-0.0025f_c' $\geq$ 0.67	α_1=0.85-0.010/1,000Xf_c' $\geq$ 0.67 β_1=0.97-0.017/1,000Xf_c' $\geq$ 0.67	0.0035
Bae and Bayrak (2003)	α_1=0.85-0.004(f_c'-70) $0.85 \geq \alpha_1 \geq 0.67$ β_1=0.85-0.004(f_c'-30) $0.85 \geq \beta_1 \geq 0.67$	α_1=0.85-0.028/1,000X(f_c'-10,000) $0.85 \geq \alpha_1 \geq 0.67$ β_1=0.85-0.028/1,000X(f_c'-4,000) $0.85 \geq \beta_1 \geq 0.67$	0.0025 for f_c' > 55 MPa (8,000 psi), otherwise 0.003
Ozbakkaloglu and Saatcioglu (2004)	α_1=0.85-0.0014(f_c'-30) $0.85 \geq \alpha_1 \geq 0.72$ β_1=0.85-0.0020(f_c'-30) $0.85 \geq \beta_1 \geq 0.67$	α_1=0.85-0.010/1,000X(f_c'-4,000) $0.85 \geq \alpha_1 \geq 0.72$ β_1=0.85-0.013/1,000X(f_c'-4,000) $0.85 \geq \beta_1 \geq 0.67$	0.003
Ibrahim and MacGregor (1997)	α_1=0.85-0.00125f_c' $\geq$ 0.725 β_1=0.95-0.0025f_c' $\geq$ 0.70	α_1=0.85-0.0086/1,000Xf_c' $\geq$ 0.725 β_1=0.95-0.0172/1,000Xf_c' $\geq$ 0.70	0.003
Azizinamini *et al*. (1994)	α_1=0.85-0.007(f_c'-69) $0.85 \geq \alpha_1 \geq 0.60$ β_1=0.85-0.008(f_c'-30) $0.85 \geq \beta_1 \geq 0.65$	α_1=0.85-0.05/1,000X(f_c'-10,000) $0.85 \geq \alpha_1 \geq 0.60$ β_1=0.85-0.05/1,000X(f_c'-4,000) $0.85 \geq \beta_1 \geq 0.65$	0.003
CEB-FIP Model Code (2010) or EN 1992 (2004)	α_1=1.0-(f_c'-50)/200 $\alpha_1 \leq 1.0$ β_1=0.80-(f_c'-50)/400 $\beta_1 \leq 0.80$	α_1=1.0-0.0345/1,000X(f_c'-7,250) $\alpha_1 \leq 1.0$ β_1=0.80-0.0172/1,000X(f_c'-7,250) $\beta_1 \leq 0.80$	(2.6 + 35 [(90 – f_c')/100]^4)/1000, f_c' in MPa

a rate of 0.04 each 10 MPa (1,450 psi) of concrete strength in excess of 70 MPa (10,150 psi). The parameter β_1 is equal to 0.85 for concrete strength up to 30 MPa (4,350 psi) and decreases linearly at a rate of 0.04 each 10 MPa (1,450 psi) of concrete strength in excess of 30 MPa (4,350 psi). Both α_1 and β_1 parameters should not be less than 0.67. It is interesting to note that the lower limits for α_1 and β_1 parameters are identical to that of CSA A23.3 (2004) recommendations.

In addition to ACI 318 (2014), Mertol *et al*. (2008), NZS 3101 (2006), CSA A23.3 (2004), Bae and Bayrak (2003), different stress block parameters have been proposed by CEB-FIP Model Code (2010), EN 1992 (2004), Ozbakkaloglu and Saatcioglu (2004), Ibrahim and MacGregor (1997), and Azizinamini *et al*. (1994). It is worth mentioning that the stress block parameters of the CEB-FIP Model Code and the Eurocode 2 (EN 1992, 2004) are identical. Table 1 summarizes stress block parameters obtained from various design codes and those from different publications. The models mentioned above are proposed for both NSC and HSC. In this research a change of the ACI provisions stress block parameters is proposed for HSC with concrete strength above 55 MPa (8,000 psi). Therefore, the proposed stress block is considered only for HSC with concrete strength above 55 MPa (8,000 psi). NSC with concrete strength below 55 MPa (8,000 psi) is taken care by the current ACI provisions. The proposal provides a

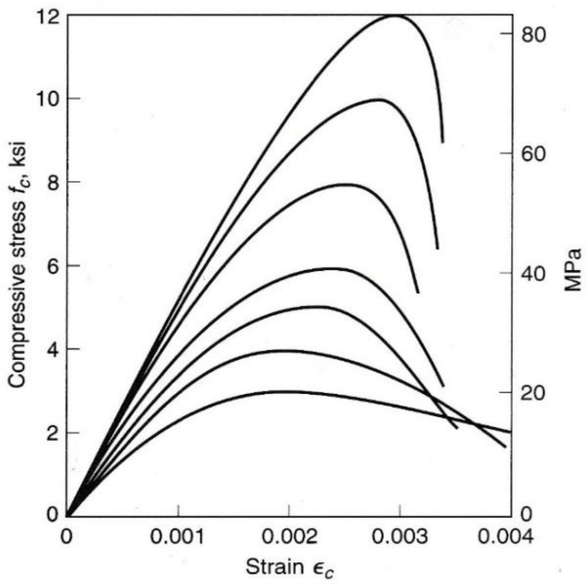

Fig. 2 Typical compressive stress-strain curves for NSC and HSC

consistent approach for determining the nominal capacity of HSC members. The combined result of the above research indicates that additional modification of the ACI rectangular stress block is needed for high-strength concrete.

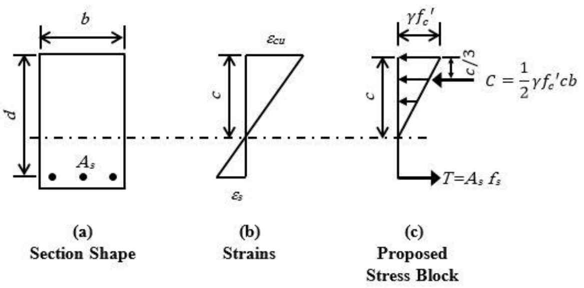

Fig. 3 Triangular stress block

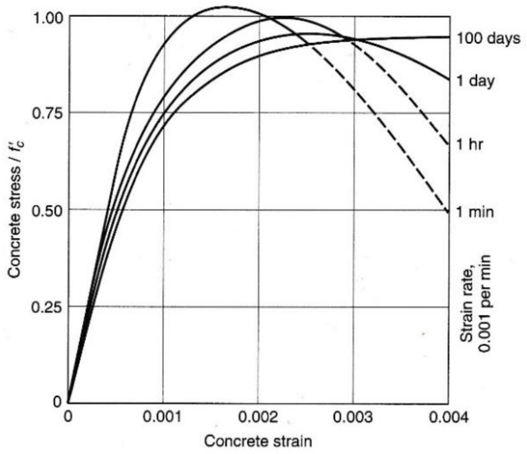

Fig. 4 Stress-strain curves at various strain rates

4. Proposed stress block model

A typical set of stress-strain curves for normal-strength concrete (NSC) and high-strength concrete (HSC) adapted from Nilson *et al*. (2010) is shown in Fig. 2. From this figure, it can be noticed that the shape of the stress-strain relationship for HSC is very close to a triangle. A triangular stress block is proposed in this research as shown in Fig. 3. This model is simple and slightly conservative since the actual area under the stress-strain curve is slightly greater than a triangle.

The curves shown in Fig. 2 are obtained from testing of concrete cylinders using uniaxial compressive tests performed at normal testing speed. The cylinder strength *fc'* is determined at normal rates of test loading. However, tests have shown that the concrete prisms and cylinders strengths under sustained loads are smaller than *fc'* (Nilson *et al*., 2010). To illustrate the effect of rate of loading, Fig. 4 that is also adapted from Nilson *et al*. (2010) shows stress-strain curves at various strain rates. A beam test may go for hours; therefore, the triangular stress block may need a reduction factor. For this purpose, a new factor (γ) is defined as shown in Fig. 3. The value of this factor can be determined by performing tests on HSC for normal load testing and sustained load testing. A value between 0.85 and 1.0 would be appropriate for evaluating test results that are conducted in a short duration. For NSC, Brachmann and Empelmann (2018) recommend a value of 0.85 for the factor (γ). They have been found that the value of 0.85 leads to a safe design. In addition, the value of 0.85 is also adapted by Danica (2016) for both NSC and HSC. Therefore, in this research, the value of the reduction factor (γ) is set to 0.85. This value is recommended until further research refines this value. If the triangular stress block is converted to an equivalent rectangular stress block, α_1 and β_1 parameters would be 0.75γ=0.64 and 0.67, respectively.

For a singly reinforced beam, equilibrium of the forces shown in Fig. 3 requires that

$$C = T \quad \text{or} \quad \frac{\gamma\ f'_c\, cb}{2} = A_s f_s \tag{1}$$

where

γ = factor used to consider the sustained load effect;
f_c' = specified compressive strength of concrete;
c = neutral axis depth;
b = breadth of the beam;
A_s = area of tension reinforcement; and
f_s = stress in tension reinforcement;

Taking moment about the resultant of the compressive force *C*, the nominal bending moment is given by

$$M_n = A_s f_s (d - \frac{c}{3}) \tag{2}$$

Making use of Eq. (1), the neutral axis depth is given by

$$c = \frac{2\ A_s f_s}{\gamma\ f'_c\, b} \tag{3}$$

For failure initiated by yielding of the tension reinforcement, the stress in tension reinforcement f_s is equal to the yielding stress f_y. Substituting Eq. (3) into Eq. (2) and knowing that the steel reinforcement ratio $\rho = A_s/bd$, yields

$$M_n = \rho b d^2 f_y (1 - \frac{2\rho\ f_y}{3\gamma\ f'_c}) \tag{4}$$

To know whether the steel has yielded at failure, the actual reinforcement ratio should be less than the balanced reinforcement ratio ρ_b. If a beam is reinforced with a tension reinforcement that is less than the balanced tension reinforcement, then the beam is classified as an under-reinforced beam. At the balanced condition, the neutral axis depth is given by

$$c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} d \tag{5}$$

where

ε_{cu} = the ultimate concrete strain at failure; and
ε_y = the yield strain of the tension reinforcement.

Substituting the value of *c* from Eq. (5) into Eq. (3), with $A_s f_s = \rho b d f_y$, one can find for the balanced reinforcement ratio

$$\rho_b = \frac{\gamma\ f'_c}{2\ f_y}\left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right) \tag{6}$$

From Table 1, it is observed that the majority of the proposed stress block models recommend a value of 0.003 for the ultimate concrete strain at failure. Therefore, the 0.003 value is retained in this research.

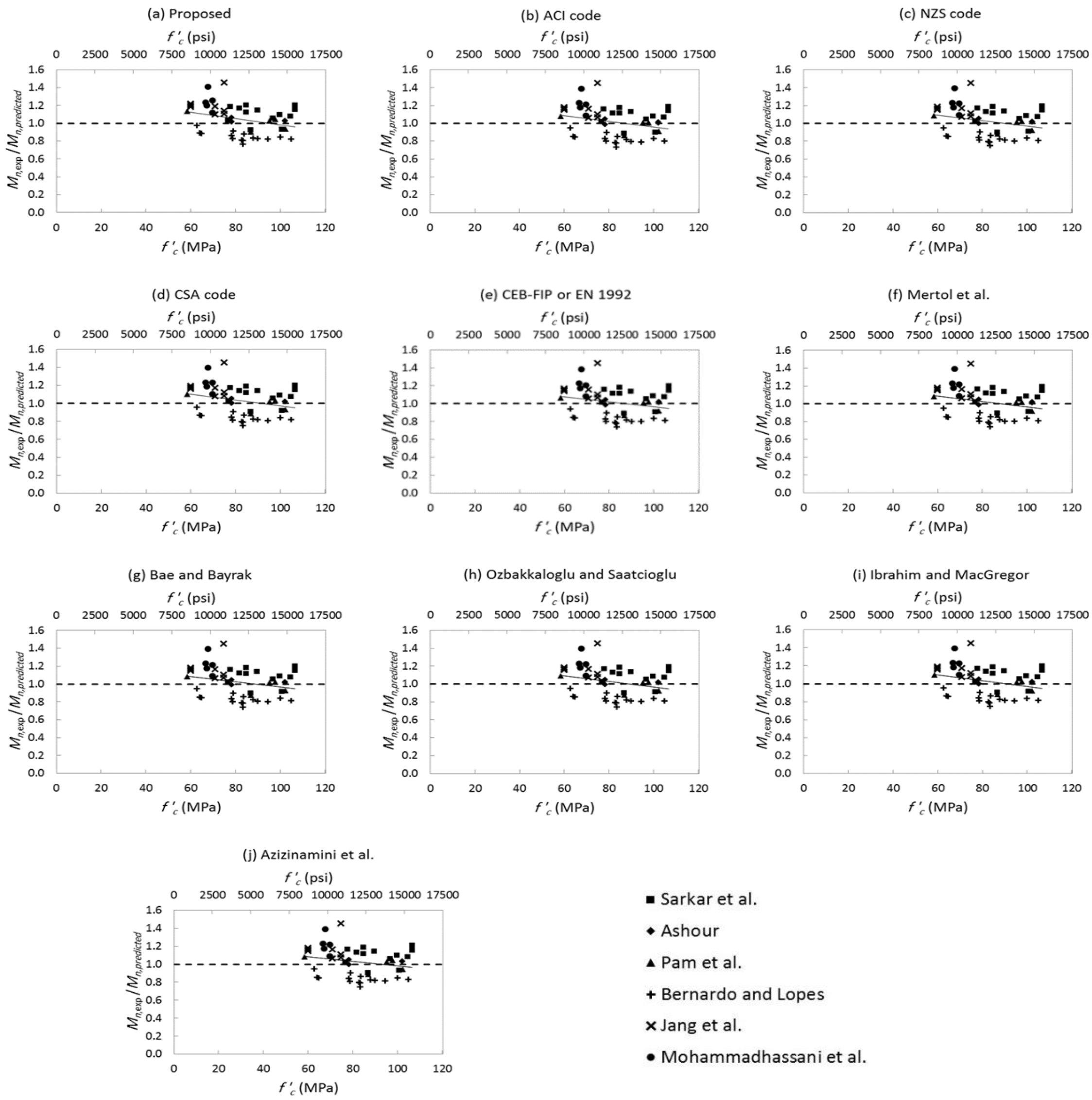

Fig. 5 Variation in experimental moment versus predicted moment capacity with compressive strength

5. Experimental validation

In this section, a validation for the nominal flexural strength for HSC beams that derived based on the proposed triangular stress block is provided. Eq. (4) is used to compare the theoretical nominal flexural strength M_n with the experimental nominal flexural strength $M_{n,exp}$ for the tested HSC beams. In addition, a comparison of the experimental moment capacity with predictions by various codes and researchers has been carried out.

5.1 Experimental data

The tested data for HSC beams considered in this study is available from Sarkar *et al*. (1997), Ashour (2000), Pam *et al*. (2001), Bernardo and Lopes (2004), Jang *et al*. (2008), Mohammadhassani *et al*. (2013). To ensure that the steel reinforcement has yielded at failure, the actual reinforcement ratio of the tested beams is compared with the balanced reinforcement ratio using Eq. (6). Some of the tested beams are excluded from consideration since they are over-reinforced beams according to Eq. (6). Since this research is focusing on HSC beams with concrete strength above 55 MPa (8,000 psi), tested beams with concrete strengths below 55 MPa (8,000 psi) are not considered. Therefore, 52 singly under-reinforced HSC beams from the overall tested data are taken into consideration. The experimental data have a concrete compressive strength between 58.6 MPa (8,500 psi) and 107 MPa (15,500 psi). The reinforcement ratio for the tested data is between 0.5% and 4.0%.

5.2 Results and discussion

The nominal moment capacities of the 52 beams were predicted by using ACI 318 (2014), NZS 3101 (2006), CSA A23.3 (2004), CEB-FIP Model Code (2010), and stress block parameters proposed by Mertol *et al*. (2008), Bae and Bayrak (2003), Ozbakkaloglu and Saatcioglu (2004), Ibrahim and MacGregor (1997), Azizinamini *et al*. (1994),

Table 2 Predictions of nominal bending moments for HSC beams: proposed and codes

Researcher(s)	$M_{n,\text{exp}}/M_{n,\text{proposed}}$		$M_{n,\text{exp}}/M_{n,\text{ACI}}$		$M_{n,\text{exp}}/M_{n,\text{NZS}}$		$M_{n,\text{exp}}/M_{n,\text{CSA}}$		$M_{n,\text{exp}}/M_{n,\text{CEB-FIP}}$	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Sarkar *et al.*	1.10	0.093	1.08	0.097	1.09	0.095	1.09	0.094	1.08	0.095
Ashour	1.03	0.018	1.01	0.022	1.02	0.020	1.02	0.020	1.01	0.023
Pam *et al.*	1.01	0.092	0.97	0.086	0.98	0.082	0.99	0.085	0.98	0.078
Bernardo and Lopes	0.85	0.051	0.82	0.051	0.83	0.049	0.84	0.050	0.83	0.048
Jang *et al.*	1.20	0.113	1.17	0.123	1.17	0.120	1.18	0.118	1.16	0.124
Mohammadhassani *et al.*	1.24	0.105	1.22	0.110	1.22	0.108	1.23	0.107	1.21	0.111
All studied beams	1.04	0.160	1.01	0.162	1.02	0.160	1.02	0.160	1.01	0.159

Table 3 Predictions of nominal bending moments for HSC beams: proposals

Researcher(s)	Ratio[(1)]		Ratio[(2)]		Ratio[(3)]		Ratio[(4)]		Ratio[(5)]	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Sarkar *et al.*	1.08	0.095	1.08	0.095	1.08	0.095	1.09	0.095	1.09	0.093
Ashour	1.01	0.022	1.02	0.022	1.02	0.021	1.02	0.020	1.02	0.023
Pam *et al.*	0.98	0.083	0.98	0.082	0.98	0.084	0.99	0.085	0.99	0.079
Bernardo and Lopes	0.83	0.050	0.83	0.049	0.83	0.050	0.84	0.050	0.84	0.047
Jang *et al.*	1.17	0.122	1.17	0.122	1.17	0.121	1.18	0.119	1.17	0.122
Mohammadhassani *et al.*	1.22	0.110	1.22	0.110	1.22	0.109	1.22	0.108	1.22	0.110
All studied beams	1.01	0.160	1.01	0.160	1.02	0.160	1.02	0.160	1.02	0.158

[(1)] Ratio of the experimental bending moment to that calculated according to Mertol *et al*.
[(2)] Ratio of the experimental bending moment to that calculated according to Bae and Bayrak.
[(3)] Ratio of the experimental bending moment to that calculated according to Ozbakkaloglu and Saatcioglu.
[(4)] Ratio of the experimental bending moment to that calculated according to Ibrahim and MacGregor.
[(5)] Ratio of the experimental bending moment to that calculated according to Azizinamini *et al*.

and the proposed equation. The predictions were compared with the tested data. Fig. 5 shows the variation of the ratio of the experimental nominal flexural moment to the predicted nominal moment with respect to the concrete compressive strength from the tested data. From this figure, it is observed that the theoretical moment strength using the proposed stress block gives a good level of conservativeness (see the trend line). However, there is a slight over estimation occurred for concrete strength above 96 MPa (14,000 psi) as shown in Fig. 5. Also, this over estimation is occurred with the predicted moments using stress block parameters proposed by various codes and proposals as shown in Fig. 5. This is happened because the predicted moment capacity for the beams tested by Bernardo and Lopes (2004) is slightly unconservative. The reason behind this is identified as the slow rate of loading and choice of demec gauges used to measure the strains along the height of the central zone of the beams. This is considered another justification for capping the concrete compressive strength by the factor (γ), which is used for the proposed triangular stress block.

The correlation of the test moment capacity versus the predicted moment capacities of the beams is shown in Fig. 6. Most of the results fall either within the ±20% band of the ideal 1:1 test moment capacity versus predicted moment capacity line, or above this band.

To see the effectiveness of the proposed triangular stress block relative to the different design codes stress block parameters, Table 2 shows the comparison between the nominal flexural strength of the tested beams with that obtained from the proposed equation and from different design codes stress block models. For all of 52 beams found in the literature, mean and standard deviation for ($M_{n,exp}/M_{n,proposed}$) ratio are 1.04 and 0.160, respectively. The proposed stress block is about at the same mean and standard deviation as compared with the NZS 3101 (2006) and the CSA A23.3 (2004) codes. However, the proposed stress block is slightly conservative. The ACI 318 (2014) provisions showed a mean value of 1.01 and a standard deviation of 0.162. The CEB-FIP model code (2010) or EN 1992 (2004) gave a mean value of 1.01 and small scatter with a standard deviation of 0.159.

Table 3 summarizes the comparison between the nominal flexural strength of the tested beams and with that obtained from different proposals by researchers. From this table, one can notice that the ability to predict the nominal moment of these proposals appears to be reasonably similar. Most of the proposals showed the same level of conservativeness and scattering as compared with each other. However, the smallest scatter is slightly better with the Azizinamini *et al*. (1994) proposal with a standard deviation of 0.158.

The mean and standard deviation for the proposed stress block are similar with that obtained from proposals of the researchers shown in Table 3. The proposed stress block is slightly conservative. The mean and standard deviation for the stress block parameters of Mertol *et al*. (2008), Bae and Bayrak (2003), Ozbakkaloglu and Saatcioglu (2004), Ibrahim and MacGregor (1997), are similar to that obtained by NZS 3101 (2006) and CSA A23.3 (2004) codes.

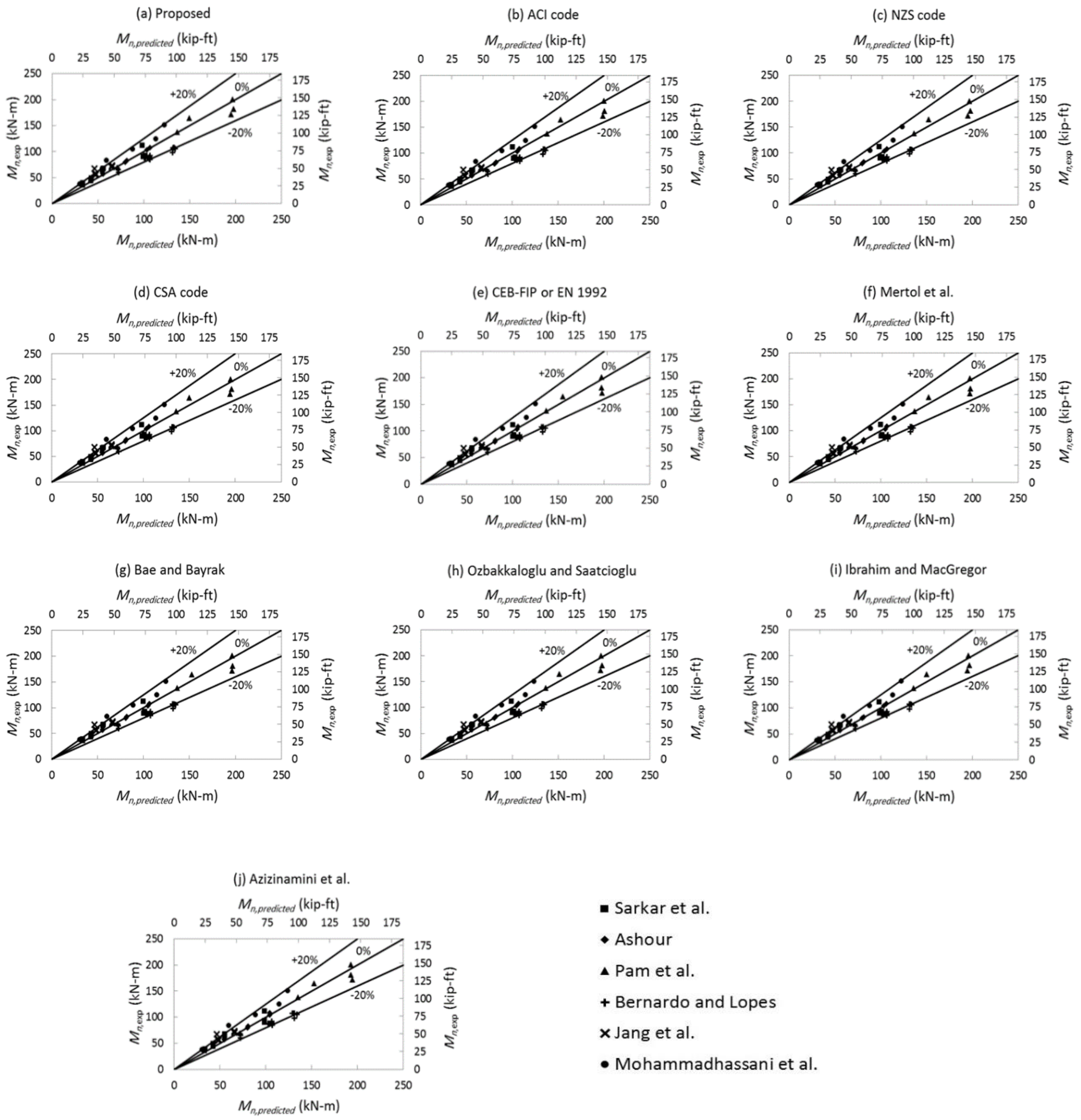

Fig. 6 Correlation of experimental moment capacity versus predicted moment capacity

The comparison showed that the proposed stress block as well as codes and proposals conservatively predict the nominal flexural strength for all the beams tested by Sarkar *et al*. (1997), Ashour (2000), Pam *et al*. (2001), Jang *et al*. (2008), and Mohammadhassani *et al*. (2013) except for the beams tested by Bernardo and Lopes (2004). It is worth mentioning that similar conclusion for HSC beams was obtained by Bae and Bayrak (2013); however, for HSC columns Bae and Bayrak (2013) concluded that the use of stress block parameters of the ACI provisions result in over-prediction of column strength. The proposed triangular stress block showed an excellent agreement with the predictions obtained by various codes and researchers proposals. In addition, the proposed triangular stress block is simpler than the stress block parameters of various codes and different methods from the literature.

The triangular stress block can be converted to an equivalent rectangular stress block. For this case, the parameters α_1 and β_1 are $0.75\gamma=0.64$ ($\gamma=0.85$) and 0.67, respectively. This would affect the ACI 318 (2014) limits for the stress block parameters α_1 and β_1. A simple change for these limits can be suggested. The β_1 can be kept as 0.65 because it is very close to the value of 0.67. Also, this parameter has a negligible effect on the nominal flexural strength of HSC beams. However, it is recommended to change the α_1 parameter from 0.85 to 0.64 for concrete strength above 55 MPa (8000 psi). However, investigation of the suitability of the proposed stress block and the modified rectangular stress block to find the axial and flexural strengths of HSC columns with concrete strength above 55 MPa (8,000 psi) is required for a future research to determine the preferred approach for HSC.

6. Future research

The proposed triangular stress block or modifications to the equivalent rectangular stress block both provide adequate prediction of beam strength for high-strength concrete. Investigation of the suitability of the proposed stress block and the modified rectangular stress block to find the axial and flexural strengths of HSC columns with concrete strength above 55 MPa (8,000 psi) is underway to determine the preferred approach for HSC.

7. Conclusions

Proposed triangular stress block is developed in this research to predict the nominal flexural strength of high-strength concrete (HSC) beams. The proposed equation and various stress block expressions were examined for estimating the nominal flexural strength of high-strength concrete beams using a data base of 52 beams available in the literature. Based on the work done in this research, the following conclusions are drawn:

1. For the tested HSC beams, the proposed triangular stress block showed a good level of conservativeness except that there is a slight over estimation occurred at a concrete strength of 96 MPa (14,000 psi) and above because the predicted moment capacity for the beams tested by Bernardo and Lopes was slightly unconservative.

2. The proposed triangular stress block showed an excellent agreement with the results obtained from various codes and proposals by researchers.

3. The proposed stress block is much simpler and slightly conservative for a wide range of concrete strengths as compared with many models proposed by codes and proposals.

4. The nominal flexural strength for HSC beams is less sensitive to the difference of stress block expressions than the ACI 318 equivalent stress block. Therefore, the ACI 318 equivalent stress block can be used to calculate the flexural strength of HSC beams.

5. Based on the proposed triangular stress block, the α_1 parameter of the ACI 318 can be changed to 0.75γ=0.64 (γ=0.85) instead of 0.85 for concrete strength above 55 MPa (8,000 psi). This is considered as a significant change to the stress block parameters for future ACI 318 provisions. However, examination of both high-strength concrete material tests for normal and sustained load and tests of high-strength concrete columns is needed to fully calibrate the factor (γ).

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