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Energy effects on MHD flow of Eyring's nanofluid containing motile microorganism

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Abstract. The impulse of this paper is to examine the influence of unsteady flow comprising of Eyring-Powell nanofluid over a stretched surface. This work aims to explore efficient transfer of heat in Eyring-Powell nanofluid with bio-convection. Nanofluids possess significant features that have aroused various investigators because of their utilization in industrial and nanotechnology. The influence of including motile microorganism is to stabilize the nanoparticle suspensions develop by the mixed influence of magnetic field and buoyancy force. This research paper reveals the detailed information about the linearly compressed Magnetohydrodynamics boundary layer flux of two dimensional Eyring-Powell nanofluid through disposed surface area due to the existence of microorganism with inclusion the influence of non- linear thermal radiation, energy activation and bio-convection. The liquid is likely to allow conduction and thickness of the liquid is supposed to show variation exponentially. By using appropriate similarity type transforms, the nonlinear PDE's are converted into dimensionless ODE's. The results of ODE's are finally concluded by employing (HAM) Homotopy Analysis approach. The influence of relevant parameters on concentration, temperature, velocity and motile microorganism density are studied by the use of graphs and tables. We acquire skin friction, local Nusselt and motil microorganism number for various parameters.

Keywords: Eyring-Powell nanofluid; variable viscosity; thermal radiation; activation energy; motile micro-organism; Homotopy analysis

1. Introduction

The essential role of energy management in the progress and formation of industrial estate is well perceived. A notable increase in the transformation of thermal energy is linked with various flow systems. Some physical executions in the manufacturing process are associated with additive thermal conductivity and heat capacity of liquids. However, large scale industries such as paper production industry, polymer extrusion industry, industry for production of chemicals rely on the basic principles of the heat transformation abilities.

Consequently, in the current age the world is taking interest to explore better ways for energy transit. One of the popular methods for this purpose is to introduce blend of metallic particles in the base liquid. This intermixing is coined as nanofluids that results in the increased thermal conductivity of based fluid. Nanoliquid is formed by the combination of small size particles of metal such as iron, titanium, gold, copper and some of their oxides are used in the base fluid that does not play key role in reaction mechanism; however they activate thermic conductivity of base fluid by stolid mode. Except this nanoliquids are commonly utilized in pharmaceutical and chemical production area. Researchers used nanofluids experimentally and theoretically in various field for large number of applications. Fluid flow behavior along the stretching cylinder in various physical conditions has gained considerable attention over the past few decades. Reasons behind the great interest are its several implementations in many advance industrial techniques

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such as composite processing, gas cooling systems, processing of plastic foam, polymer technology, watering system channels, cement process industry and many more. Primarily, (Choi 1995) introduced the word “nanofluid”, Then Brownian movement and thermophoretic limitations were considered sufficient for the development of nanofluid model. This theory demands the scientists to show few engrossing flow features for different nanofluids in the existence of various fluid models.

Model of Eyring-Powell was deduced through kinetic theory of fluids. (Gireesha *et al*. 2015) showed three dimensional MHD flow Eyring-Powell liquid along radiation of heat and convective boundary conditions passing through stretched surface area. In addition to it, they under took the impacts of sling nano particles and heat generation/absorption. (Akbar *et al*. 2015) inquire the purpose the Eyring-Powell liquid showing properties of magnetic field over stretched surface. (Timol and Patel 2009) took advantage of asymptotic border state to reveal the concept of Eyring-Powell liquid. (Araa *et al*. 2014) showed Eyring-Powell liquid with the help of radiation over the stretch surface. (Ghajar and Yoon 1987) explore the Eyring-Powell fluid equation and observe its differences in rate of share viscosity. (Waqas *et al*. 2017) observed the somatic nature of Cattaneo-Christov scattering effects on Eyring-Powell fluid for thermal dependent conduction. (Khan *et al*. 2017) observed Eyring-Powell fluid of MHD have movement through a plate in the existence of heterogeneous-Homogeneous retort. Muthtamilselvan *et al.* (2019) studied an inclined magnetohydrodynamic flow of a micropolar fluid occurring between two stretchable disks rotating co-axially at a constant distance apart by taking into account higher order chemical reaction effects. Using similarity variables the Navier–Stokes equations, which represent the momentum, microrotation, energy, and concentration, are transformed into ordinary differential equations. The transformed conservation equations are solved numerically by using the Nachtsheim–Swigert shooting iteration technique along with sixth-order Runge–Kutta integration scheme.

The Bio-convection flow is stimulated due to floating of motile small living micro-organisms which are heavier then water (Eldabe *et al*. 2020). The self- propelling small living microorganisms escalate the density of basic liquid in particular direction. The group of small living organism in the above surface forms suspended solutions to thick that results uncertainty. Under such conditions, Convection uncertainty and production of convection phenomenon takes place. Irregular and swift movement of small living organism results in production of bio-convection. Bio-convection uncertainty is evolving by an initial consistent suspension in the absence of unbalanced disturbance of density that was observed by (Pedley *et al*. 1988). Muthtamilselvan and Renuka (2018) studied the nanofluid flow and heat transfer induced by two co- axially rotating disks using Buongiorno’s model. This model took into account the Brownian diffusion and thermophoresis effects due to the presence of nanoparticles. The governing partial differential equation was transformed into a set of nonlinear ordinary differential equations by using similarity transformation and solved numerically using shooting techniques.

Many scientist worked on bio convection with various configurations are mentioned in the references (Pal *et al*. 2019, Kuznetsov 2006, Kumar *et al*. 2019). In bioscience fuid mechanics, latest important developing areas are flow inclusive nano bio-convection sling with small size particles and base liquids. The nano particles are not self-propelling such as motile small living organism, the movement of nano particles by reason of Brownian motion and thermophoresis if the cluster of nano particles are small in size phenomenon of bio-convection is takes place in nanofluid. The latest advancement of nano bio-convection compresses small living organism is indicated in references (Eldabe *et al*. 2020, Mehryan *et al*. 2016, Belabid and Allali 2019, Bhatti and Michaelides 2020, Balla *et al*. 2020). Al-Amri and Muthtamilselvan (2020) focused on the magnetohydrodynamic nanofluid containing micro-organisms flow in the subsistence of thermophoresis and Brownian action event. Furthermore, Dufour and Soret effects have been taken into detail through the heat and mass transfer analysis. The coordination of partial differential equations renewed into a scheme of ordinary differential equations using the similarity renovation and also numerically solved via bvp4c MATLAB package. The physical parameters are discussed through different graphs and tabular form. Moreover, it is exposed that the raising concentration and temperature with greater utility of Dufour and Soret effect.

Thermal radiation is a technique for transfer of heat which is an interesting and important notion for scientists due its applications in combustion, solar ponds, nuclear power plants, furnace design, and reactors of photochemical. Turbines, solar collections, are used in various fields of engineering. For example, space vehicles, satellites, missiles. Detailed studies in this topic ate described in the publications (Bhatti *et al*. 2019, Ghdikolaei and Gholinia 2019, Ghdikolaei and Gholinia 2019, Li *et al*. 2019, Li *et al*. 2019). Impact of heat transfer on incompressible, steady flow of water base nano liquid over a stretchable object due to transverse magnetic field along buoyancy effects and thermal radiation are discussed by (Rashidi *et al*. 2014). Influence on non-consistent magnetic field in forced convection transfer of heat of nanoliquid has been examined by (Sheikholeslami *et al*. 2015). Free convection observation of magnetic nanoliquid including viscidity characteristic has been discussed by (Sheikholeslami *et al.* 2016). Ramya *et al.* (2018) developed a mathematical model to examine the effects of radiation and slanted magnetic on boundary layer flow of a micropolar fluid containing gyrostatic microorganisms through a vertical fixed or continuous moving porous plate. The governing boundary layer equations are cast into a matrix form and solved analytically by utilizing the state space approach and the inversion of the Laplace transform is carried out, utilizing numerical approach. Numerical outcomes for the momentum, microrotation, density of motile microorganism and temperature distributions are given and illustrated graphically for the problem.

The concept of activation energy was first governed in 1889 by Svante Arrhenius, has successfully enhanced the engrossment of researchers and engineers. In actual, the activation energy is a basic energy that is distributed among the reactants for changing into final production in different chemical reactions. The potential and kinetic energy linked with the molecule are effective for separation of bond or to extend and bent the bond. Molecules bounce back with every one without accumulation of chemical reaction if their motion is slack down with less amount of kinetic energy. Although in the existence of large momentum energy chemical reaction occurs with less energy activation. Hydrodynamics, oil storage industries, suspension of oil, the idea of energy activation is very important due to large number of applications, many researcher observed this aspect. For example, (Maleque 2013) examined the impact of exothermic/endothermic reactions on the top of flat plate in the presence of porous medium and activation energy. An irregular flow of radiative of nanoliquid with the effects of energy activation and cross diffusion was considered by (Khan *et al*. 2018). Flow of a liquid in circulating frame due to the energy activation was examined by (Shafique *et al*. 2016). Another comparative observation was discussed by (Awad *et al*. 2014) explain the impact of energy activation by the use of viscid fluid model. A significant participation related to the use of activation energy in contribution with respect to the utilization of energy activation in thermal exclusion structure by the use of non-Newtonian nanoliquid was observed by (Hsiao 2017).

A glance at the literature reveals that limited studies have done on peristaltic flow of bio-convection by regarding Eyring–Powell nanoliquid due to porous medium and magnetic field. Although, (Akbar 2015) has investigate the peristaltic flow of bio-convection including gyrotactic small living organism in a systematic way. (Bhatti *et al*. 2017) observed the non- Newtonian peristaltic flow Jefrey nano liquid in the existence of condensation and non-uniform magnetic field comprises gyrotactic small organism in annular. However, Peristalsis is familiar structure for the transportation of physiological liquid in many biotic organs. Biotic structure are considered to be non-uniform, we studied freebie convection peristaltic transportation in a non-consistent way fill up by Eyring–Powell nano liquid comprises gyrotactic small organism. The recent observation has large number of applications in engineering and medical sciences. However, small organism is useful in producing oxygen, decaying of organic material and sustaining of human health. By reducing the concentration of Small organism in the nanoliquid brings change in thermal conductivity. Several researchers used different approaches for the investigation of frequency of cylinders and concrete material (Kagimoto *et al.* 2015, Mesbah and Benzaid 2017, Alijani and Bidgoli 2018, Demir and Livaoglu 2019, Samadvand and Dehestani 2020).

The main objective of present study is to investigate the influence of unsteady flow comprising of Eyring-Powell nanofluid over a stretched surface. This research paper reveals the detailed information about the linearly compressed Magnetohydrodynamics boundary layer flux of two dimensional Eyring-Powell nanofluid through disposed surface area due to the existence of microorganism with inclusion the influence of non- linear thermal radiation, energy activation and bio-convection. This work aims to explore efficient transfer of heat in Eyring-Powell nanofluid with bio-convection. Nanofluids possess significant features that have aroused various investigators because of their utilization in industrial and nanotechnology. The influence of including motile microorganism is to stabilize the nanoparticle suspensions develop by the mixed influence of magnetic field and buoyancy force. This method converge fastly than other methods. It is keenly seen from the literature, no evidence is found concerning current model. By using appropriate similarity type transforms, the nonlinear PDE's are converted into dimensionless ODE's. The results of ODE's are finally concluded by employing (HAM) Homotopy Analysis approach. The influence of relevant parameters on concentration, temperature, velocity and motile microorganism density are studied by the use of graphs and tables. We acquire skin friction, local Nusselt and motil microorganism number for various parameters. In view of all aforementioned applications, the purpose of our current investigation is to evaluate the two-dimensional bio-convection flow of Eyring-Powell nanoliquid with a stretched surface. The joint influence of magnetic field and heat source/sink are also discussed. The dimensionless boundary-value problem is resolved analytically by the method of Homotopy Analysis (Renuka *et al*. 2020, Alamri *et al*. 2019). The effects of various parameters are also examined with the help of tables and graphs. This research relates to a practicing heat transfer engineer as in the three conservation laws: mass, energy, and momentum. Both are primarily concerned with energy-related subject matter and both, in a very real sense, supplement each other. However, thermodynamics deals with the transfer of energy and the conversion of energy into other forms of energy (e.g., heat into work), with consideration generally limited to systems in equilibrium. The topic of heat transfer deals with the transfer of energy in the form of heat; the applications almost exclusively occur with heat exchangers that are employed in the chemical, petrochemical, petroleum (refinery), and engineering processes.

2. Mathematical formulation

Assume an incompressible and steady outflow of Eyring–Powell nanoliquid which is incompressible, two dimensional over a stretched sheet containing microorganism, thermal radiation, heat source/sink and activation energy. A sturdy magnetic field is introduced in the normal way. Where u is a velocity component with x-axis and v is a velocity component with y-axis, $v = \frac{\mu}{\rho}$ denotes the kinematic viscosity, μ represents dynamic viscidity, $(\rho)_f$ is base fluid density, g denotes the gravitational acceleration, D_B is Brownian diffusion coefficients, D_T is the thermophoresis diffusion coefficients, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ denotes the ratio of effective heat capacity to the nanoparticles material to the heat capacity of liquid. D_m is diffusion of motile microorganism, W_c denotes highest speed of cell-swimming, $K_{c^*}(C - C_\infty)\left(\frac{T}{T_\infty}\right)^n exp\left(-\frac{F_a}{kT}\right)$ is Arrhenius expression, F_a is energy activation and is reaction rate K_{c^*}.

The governing equations for Eyring-Powell bioconvectional nanofluid (Gireesha *et al*. 2015; Akbar *et al*. 2015, Timol and Patel 2009, Araa *et al*. 2014, Ghajar and Yoon 1987, Waqas *et al*. 2017, Khan *et al*. 2017) are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\Gamma\begin{pmatrix}u^2\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 u}{\partial y^2}\\ +2uv\frac{\partial^2 u}{\partial x\partial y}\end{pmatrix}= \left(\nu+\frac{1}{(\rho)_f\beta c}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\left(-\mu\varsigma\frac{\partial T}{\partial y}\right) -\frac{1}{6(\rho)_f\beta c^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\frac{\sigma u B^2}{a\rho}, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma_s T^3{}_\infty}{3K^\circ(\rho c)_f}\right)\left(\frac{\partial^2 T}{\partial y^2}\right)+ \frac{Q}{(\rho c)_f}(T-T_\infty)+\tau\begin{Bmatrix}D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)\\ +\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\end{Bmatrix} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)- K_{c^*}(C-C_\infty)\left(\frac{T}{T_\infty}\right)^n exp\left(\frac{-E_a}{kT}\right) \quad (4)$$

$$u\left(\frac{\partial n}{\partial x}\right)+v\left(\frac{\partial n}{\partial y}\right)+\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right] = D_m\left(\frac{\partial^2 n}{\partial y^2}\right), \quad (5)$$

The related physical boundary conditions are

$$u=u_w(x)=ax, v=0, T=T_w, C=C_w, n=n_w, \text{at } y=0, \quad u\to 0, T\to T_\infty, C\to C_\infty, n\to n_\infty, \text{ at } y\to\infty, \quad (6)$$

Where T is liquid temperature, T_w is temperature of wall and T_∞ is liquid temperature at free stream. C is concentration of liquid, C_w is stretching sheet concentration of and C_∞ is liquid concentration at free stream. n_w, n_∞ is motile organisms and motile organisms of liquid at free stream.

The expansion in viscidity with temperature is expressed as

$$\mu=\mu_0 e^{-\zeta(T-T_\circ)}, \quad (7)$$

Where ς is steady.

Let us introduce the following appropriate transformation are

$$\eta=\sqrt{\frac{a}{\nu}}y, \psi=\sqrt{a\nu}xf(\eta), g(\eta)=\frac{T-T_\infty}{T_W-T_\infty}, j(\eta)=\frac{C-C_\infty}{C_W-C_\infty}, \chi(\eta)=\frac{n-n_\infty}{n_W-n_\infty}, u=axf'(\eta), v=-\sqrt{\nu a}f(\eta), \quad (8)$$

Where$\eta$is the similarity variable, $f(\eta)$ the dimensional less stream function, $g(\eta)$ the non-dimensional temperature function, $j(\eta)$ the non-dimensional concentration and $\chi(\eta)$ the non-dimensional microorganism function. Using relations (8), the Eq. (1) is satisfied. Eq. (2) to Eq. (6), yield as below

$$(1-Ag)(f'''-Af''g')+ff''+Mf'''- M\lambda f'''f'^2--Haf'-f'^2+\Gamma[g-Nrj-Nc\chi]=0 \quad (9)$$

$$\frac{1}{Pr}g''+fg'+Nbg'j'+Ntg'^2+Rdg''+\beta g=0 \quad (10)$$

$$j''+Pr\,Lefj'+\frac{Nt}{Nb}g''-Pr\,Le\delta(1+\omega g)^m exp\left(\frac{-E}{1+\omega g}\right)j=0, \quad (11)$$

$$\chi''+Lbf\chi'-Pe(j''(\chi+\Omega)+\chi'j')=0, \quad (12)$$

Non-dimensional forms of boundary conditions are

$$f(0)=0, f'(0)=1, g(0)=1, j(0)=1, \chi(0)=1, \quad f'(\infty)\to 0, g(\infty)\to 0, j(\infty)\to 0, \chi(\infty)\to 0. \quad (13)$$

Where $M=\frac{1}{\mu\beta c}$ and $\lambda=\frac{a^3x^2}{2c^2\upsilon}$ are liquid parameters, $Ha=\frac{\sigma B^2}{a(\rho)_f}$ is magnetic range, $Pr=\frac{\mu}{\alpha}$ denotes the Prandtl number, $Nt=\frac{\tau D_T(T_W-T_\infty)}{T_\infty\upsilon}$ denote the thermophoresis parameter, $N_b=\frac{\tau D_B(C_W-C_\infty)}{\nu}$ is Brownian motion constraint, $L_e=\frac{\alpha}{D_B}$ Lewis number, $A=-\varsigma(T_w-T_\infty)$ represent the variable viscidity parameter, $\alpha=\frac{\beta g(1-C_\infty)(T_W-T_\infty)}{a^2x}$ mixed convection parameter, $Nr=\frac{(\rho_P-\rho_f)(C_W-C_\infty)}{\beta\rho_f(1-C_\infty)(T_f-T_\infty)}$ denote the buoyancy number, $Nc=\frac{\gamma(n_w-n_\infty)(\rho_m-\rho_f)}{\beta\rho_f(1-C_\infty)(T_f-T_\infty)}$ bio-convection Rayleigh number, $R=\frac{16\partial_s T_\infty{}^3}{K^\circ K}$ thermal radiation constraint, $Pe=\frac{bW_c}{D_m}$ is Peclet number, $\Omega=\frac{n_\infty}{(n_W-n_\infty)}$ denote the motile micro-organism concentration difference constraint, $F=\frac{E_a}{kT_\infty}$ is activation energy, $\omega=\frac{(T_f-T_\infty)}{T_\infty}$ is differences of temperature parameter, $\delta=\frac{K_c{}^2}{a}$ is reaction rate.

The significant physical abundance i.e. skin friction, Nusselt ,Sherwood numbers and motile microorganisms are express as

$$C_f=\frac{\tau_w}{\rho(ax)^2}, Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \quad S\hbar_x=\frac{xq_m}{D_B(C_W-C_\infty)}, \; Nn_x=\frac{xq_n}{D_B(N_W-n_\infty)},$$

Here $\tau_w=\left[\left(\nu+\frac{1}{\rho\beta c}\right)\frac{\partial u}{\partial y}-\frac{1}{6\beta c^3}\left(\frac{\partial u}{\partial y}\right)^3\right]_{y=0}$ is the exterior shear stress with x-axis, $q_m=-D_B\left(\frac{\partial c}{\partial y}\right)_{y=o}$ is the exterior mass flux, $q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ the heat flux. The non-dimensionl form of skin friction, Nusselt number, Sherwood numbers and motile micro-organisms are expressed as

$$C_f\,Re_x{}^{1/2}=\left[(1+M)f''(o)-\frac{\lambda}{3}Mf''^3(0)\right],$$

$$Nu_x\,Re_x{}^{-1/2}=-g'(0),$$

$$S\hbar_x\,Re_x{}^{-1/2}=-j'(0),$$

$$Nn_x\,Re_x{}^{-1/2}=-\chi'(0),$$

Where the Reynold number $Re_x{}^{1/2}$ is expressed as

$$Re_x{}^{1/2}=\left(\frac{ax^2}{\nu}\right)^{\frac{1}{2}}.$$

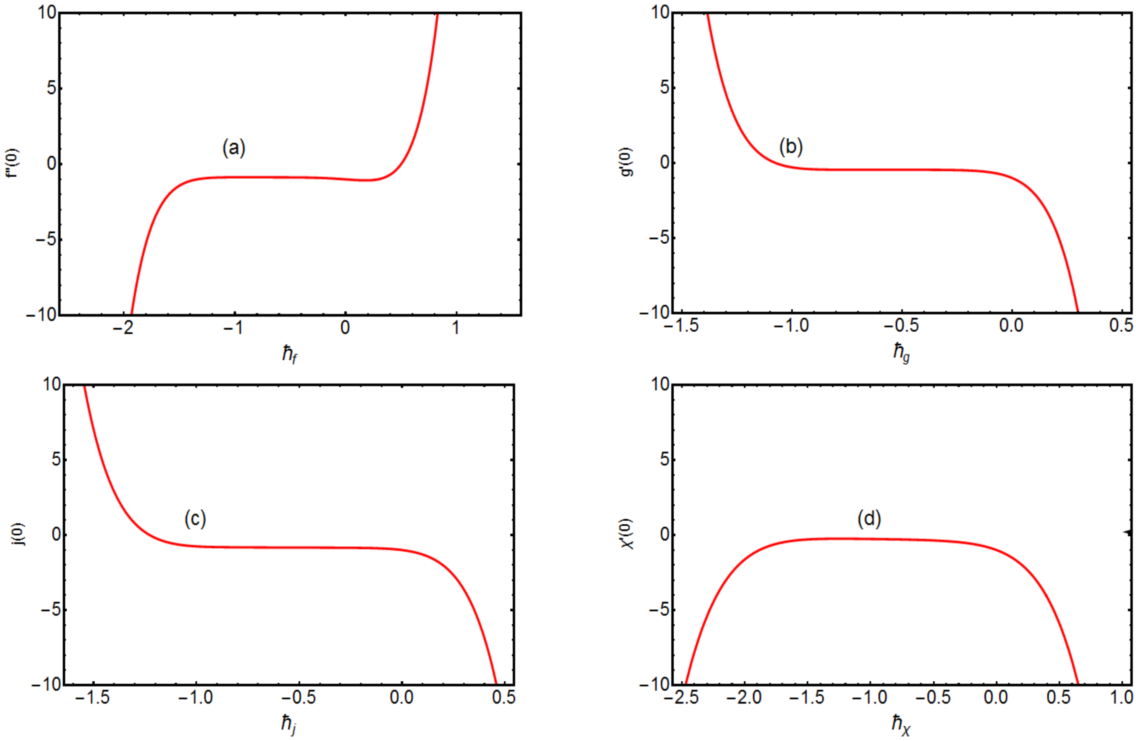

Fig. 1 Convergent h curves of (a) f, (b) g, (c) j, (d) χ

Table 1 Convergence of $f''(0), g'(0), j'(0)$ and $\chi'(0)$ by HAM for various orders of approximations when $Pr=Lb=Ha=Pe=\Gamma=1$, $M=\lambda=Nt=Nb=Rd=\omega=\Omega=\delta=A=E=1/10$, $Nr=Nc=0.2$

Order of approximation	$-f''(0)$	$-g'(0)$	$-j'(0)$	$-\chi'(0)$
1	0.750027	0.678684	0.687484	0.976729
5	0.738671	0.676627	0.701838	1.017260
11	0.743162	0.674431	0.735442	1.053831
15	0.743963	0.674442	0.716739	1.032180
18	0.739344	0.674421	0.715638	1.032210
20	0.749596	0.674420	0.715623	1.032208
24	0.749599	0.674418	0.715621	1.032201

3. The analytic solution

Homotopy Analysis approach is logical estimate for non-linear differential equations, because different problems soaring in technology and science fields are non-linear particular in fluid mechanics. Ordinary differential equations (non-linear) (9)-(12) with the boundary conditions (13) are worked out analytically by applying Homotopy analysis technique. Here $f_o(\eta), g_o(\eta), j_o(\eta)$ and $\chi_o(\eta)$ are initial guesses and linear operators are L_f, L_g, L_j and L_χ. For such analytic result correspond to the transfer of heat and mass, the well-suited initial guesses and linear operators are given below

$$f_0(\eta) = 1 - e^{-\eta}, g_0(\eta) = e^{-\eta}, j_0(\eta) = e^{-\eta},$$
$$\chi_0(\eta) = e^{-\eta},$$
$$L_f = f''' - f', L_g = g'' - g, L_j = j'' - j,$$
$$L_\chi = \chi'' - \chi,$$

With

$$L_f(c_1 + c_2 e^{\eta} + c_3 e^{-\eta}) = 0, L_g(c_4 e^{\eta} + c_5 e^{-\eta}) = 0,$$
$$L_j(c_6 e^{\eta} + c_7 e^{-\eta}) = 0, L_\chi(c_8 e^{\eta} + c_9 e^{-\eta}) = 0.$$

Where c_1, c_2 and c_9 are arbitrary constants.

4. Convergence analysis

Solution of Homotopy Analysis method firmly based on values of auxiliary parameter. The assisting parameters $\hbar_f, \hbar_g, \hbar_j$ and $\hbar_\chi$ have vital role in adjusting and convergence for the series solution. Fig. 1 represent the h curves, and found that the appropriate ranges of the auxiliary parameters are $-1.2 \le \hbar_f \le 0.1$, $-1.0 \le \hbar_g \le -0.1$, $-1.0 \le \hbar_j \le -0.1$ and $-1.6 \le \hbar_\chi \le -0.1$. Convergence series of momentum, temperature, concentration and density equations are shown in Table 1. It is observed that order of estimate at 15th iteration is noticed for convergence of $g'(0)$ whereas convergent solution of $f''(0), j'(0)$ and $\chi'(0)$ is acceptable at the 24th order of approximation.

5. Analytic results and ciscussion

This reasonable section analyzes the results flow of two-dimensional of Eyring-Powell nanoliquid on a stretching

Table 2 Numerical data of effective skin friction for various parameters

M	A	Nr	Nc	λ	Ha	$-C_f \mathrm{Re}_x^{1/2}$
0.1	0.1	0.1	0.1	0.1	0.1	1.1128
0.2						1.1662
0.3						1.2184
	0.2					1.1128
	0.4					1.1129
	0.6					1.1128
		0.2				1.1307
		0.5				1.1477
		1.0				1.1765
			0.2			1.1393
			0.5			1.1632
			0.6			1.1668
				0.1		1.1393
				0.3		1.1444
				0.5		1.1454
					0.0	1.1247
					0.5	1.1252
					1.0	1.1250

Table 3 Numerical data of Nusselt number for distinct parameters

Pr	Nb	Nt	Rd	$-g'(0)$
1.0	0.2	0.2	0.2	0.5673
1.2				0.6065
1.6				0.6358
	0.3			0.5321
	0.5			0.4645
	1.0			0.3374
		0.1		0.5686
		0.2		0.5673
		0.3		0.5639
			0.1	0.5452
			0.2	0.5673
			0.3	0.5948

Table 4 Numerical data of motile microorganism for distinct parameters

Γ	Nc	Pr	Pe	$-\chi'(0)$
0.2	0.1	1.0	0.5	0.5213
0.3				0.5516
0.4				0.5674
	0.2			0.5115
	0.4			0.5091
	0.6			0.4913
		1.0		0.5213
		1.5		0.7044
		2.0		0.7380
			0.5	0.5213
			1.0	0.5180
			1.5	0.5073

sheet that consisting motile microorganisms in the existence of thermic radiation and effects of heat source/sink. Homotopy analysis approach assures the convergent series solution. It is noticed that analytically calculated solutions

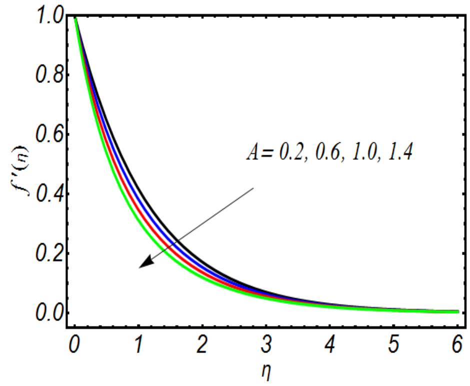

Fig. 2 Impact of A on $f'(\eta)$

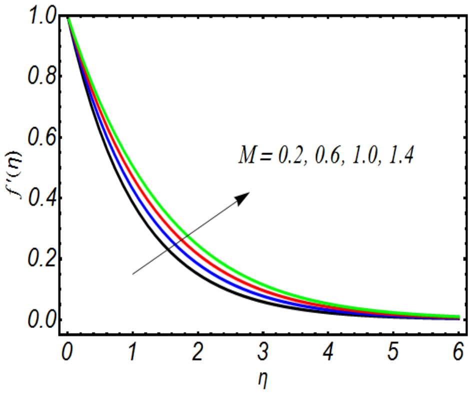

Fig. 3 Impact of M on $f'(\eta)$

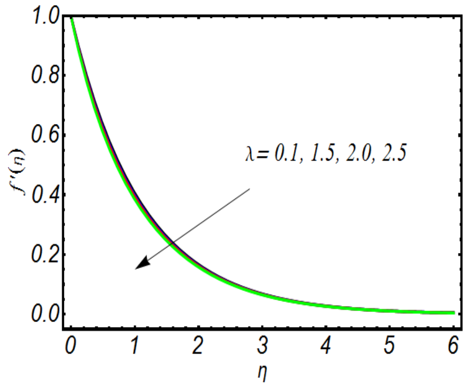

Fig. 4 Impact of λ on $f'(\eta)$

for the velocity, temperature, concentration and microorganism fields are exhibited graphically, in order to obtain definite perception of the problem, for different values of the parameter A, fluid parameter M and λ. Ha magnetic field parameter, buoyancy ratio parameter Nr, Rayleigh number Nc, Γ the relation of buoyancy force to viscidity force, Prandtl number Pr, parameter of Brownian motion Nb. Thermophoresis parameter Nt, parameter of thermal radiation Rd, heat source/sink S, Lewis number Le, activation energy E, bio-convected Lewis number Lb, bio-convection Peclet number Pe appearing in rescaled equations of the problem.

Figs. 2-8 reveal the influence of various factors on velocity of liquid. Fig. 2 shows that by increasing the variable viscidity factor A, velocity shows changes in

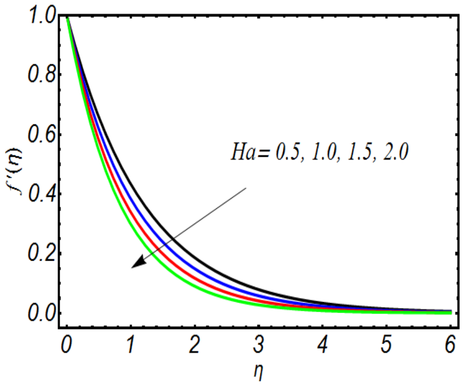

Fig. 5 Impact of Ha on $f'(\eta)$

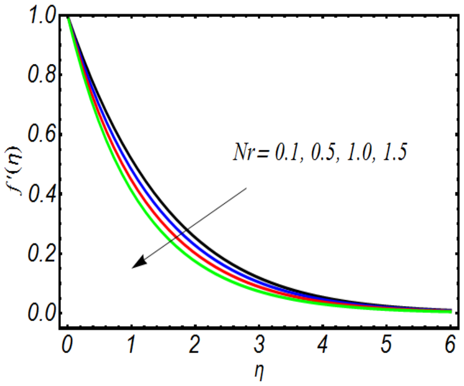

Fig. 6 Impact of *Nr* on $f'(\eta)$

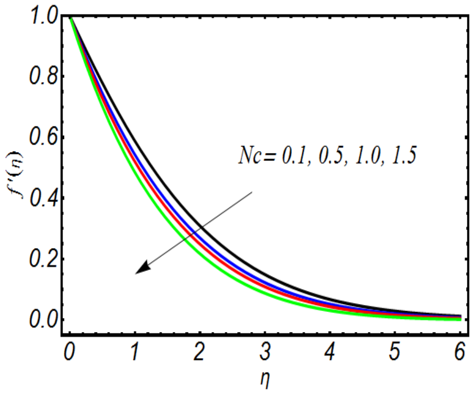

Fig. 7 Impact of *Nc* on $f'(\eta)$

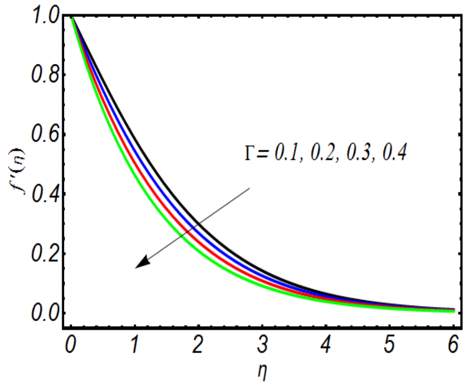

Fig. 8 Impact of Γ on $f'(\eta)$

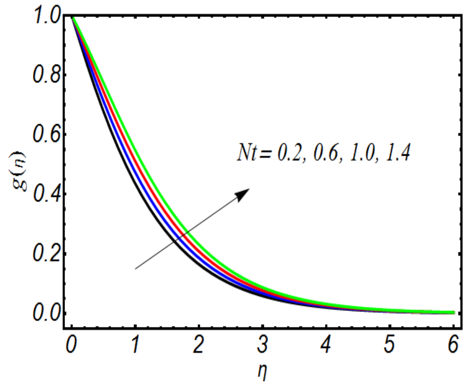

Fig. 9 Impact of *Nt* on $f'(\eta)$

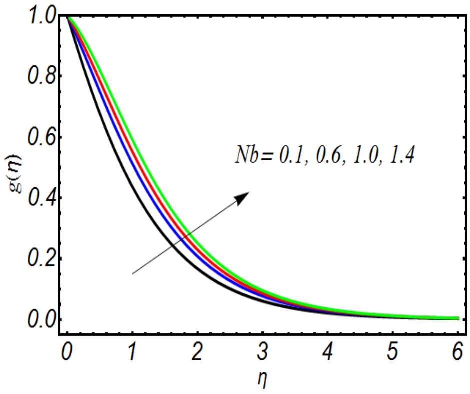

Fig. 10 Impact of *Nb* on $f'(\eta)$

decreasing order. However by enhancing the value of *A*, viscidity of fluid in increasing order causes decrease in velocity. Figs. 3 and Fig. 4 respectively reveal the influence of Eyring-Powell fluid factor λ and magnetic parameter *M* on $f'(\eta)$, the non-dimension velocity. By enhancing fluid factors *M* velocity is increased. As we known fluid factor *M* has contrary relation with viscidity. However for greater value of λ, the liquid shows thickness which is the reason for decreasing in velocity of fluid. Fig. 5 shows that for greater magnetic range *Ha* the Lorentz force shows greater strength which increases the resistance to fluid flow, so velocity decreases. In Figs. 6-7 shows when the values of buoyancy ratio parameter *Nr* and Rayleigh number *Nc* enhance then decreasing velocity order is noticed. However, by increase in value of *Nc* the buoyancy force due to phenomenon of bio-convection results in decrease of velocity. The effect of Γ on f' is express in Fig. 8. We know Γ shows the relation of buoyancy force to viscidity force. The enhancement in the value of buoyancy force enhancing the value of Γ, which results a decrease in the fluid velocity. Figs. 9-14 reveal the change in temperature for various values of Prandtl number *Pr*, Brownian motion *Nb*, Thermophoresis parameter *Nt*, thermal radiation *Rd*, heat source/sink β. Fig. 9 shows that by enhancing thermophoresis factor, temperature enhances due to thermophoresis in which tiny size particles move away from warm surface area to cold surface area and it causes a rise in temperature. Same behavior is discussed in (Saleem *et al*. 2019). Fig. 10 shows that by enhancing Brownian motion irregular movement of liquid particles enhances as a result

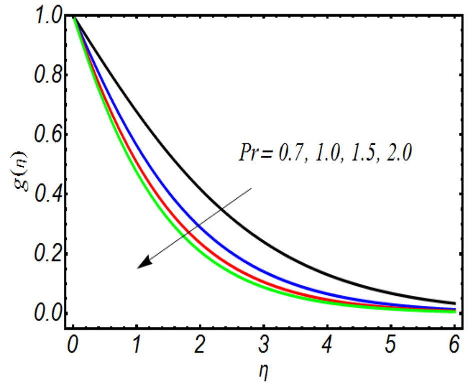

Fig. 11 Impact of Pr on $g(\eta)$

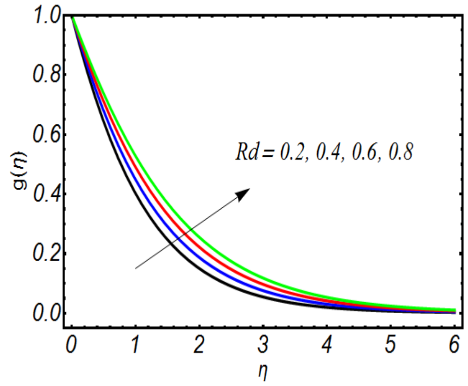

Fig. 12 Impact of Rd on $g(\eta)$

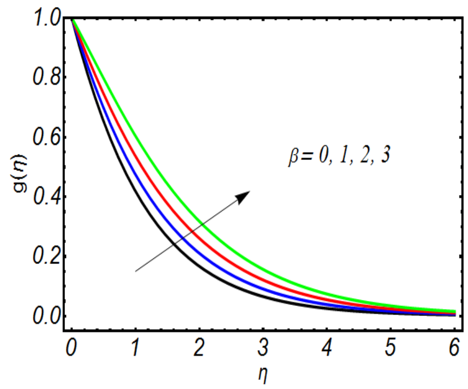

Fig. 13 Impact of β on $g(\eta)$

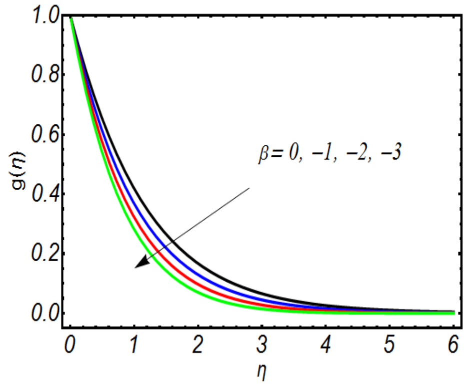

Fig. 14 Impact of β on $g(\eta)$

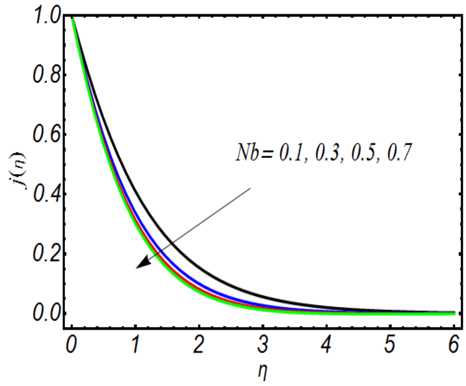

Fig. 15 Impact of Nb on $j(\eta)$

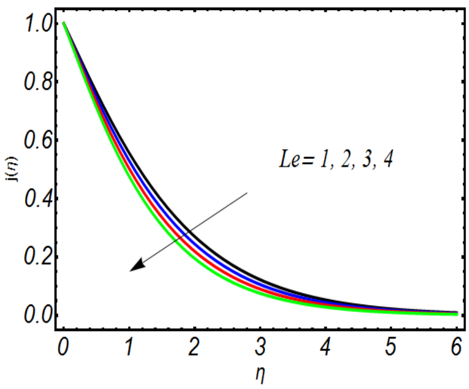

Fig. 16 Impact of Le on $j(\eta)$

the temperature is increased. Effect of Prandtl number Pr on temperature field is demonstrated in Fig. 11. The greater values of Pr shows decrease in temperature parameter as $Pr = \frac{\nu}{\alpha}$, hence thermal conductivity reduces by enhancing Pr so temperature decline. By viewing diagram Fig. 12, it is noticed that distribution of temperature and thickness of linked thermal boundary layer enhance for greater value of radiation parameter. It happens due to reason that heat flux of surface enhances by the influence of thermal radiations that causes enhance in temperature, within the boundary layer. The influence of thermal source $\beta>0$ and absorption of heat $\beta<0$ on temperature can be observed in Figs. 13-14, we observed that thickness of boundary layer and temperature increase for heat source in contrast to the results of heat absorption.

Figs. 15-17 show the fluctuation in concentration profile for various values of Lewis number Le, Brownian motion Nb and activation energy E. Fig. 15 shows that by rising Brownian motion Nb the rate of mass removal decrease due to by increasing the value of Brownian motion Nb the random movement of nano particles enhances which decrease the rate of mass transfer. Fig. 16 shows the effect of Lewis number Le on concentration field. It is observed that by increasing Lewis number the volume friction of nano particles increase that decreases the concentration due to boundary layer thickness. Fig. 17 shows concentration profile is enhanced by increase in value of non- dimensional activation energy E that shows the greater concentration of thickness for boundary layer. Physically greater activation energy E and low temperature shows less rate of reaction.

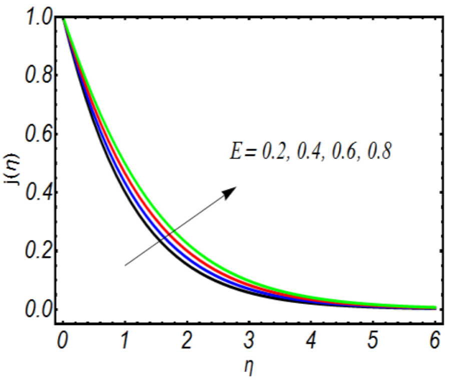

Fig. 17 Impact of E on $j(\eta)$

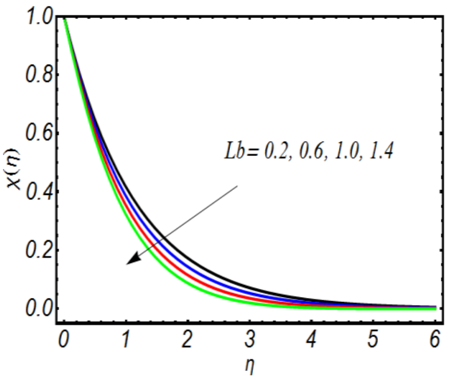

Fig. 18 Impact of Lb on $\chi(\eta)$

That result in reducing the rate of chemical reaction.

Figs. 18-19 show the influence of bio-convected Lewis number Lb, bio-convected Peclet number Pe on microorganism $\chi(\eta)$. From Fig. 18, a reducing trend is observed by the increase in value of Lb, this mode is observed due to less diffusivity of microorganism. The less diffusivity occur for increased Lb due to which $\chi(\eta)$ is decreased. In Fig. 19, it is examined that $\chi(\eta)$ is reduced with enhance in bio-convected Peclet numberPe. Hence the large speed of cell-swimming is increased with an increase in Pe. This large speed of cell-swimming is the reason in the less behavior of $\chi(\eta)$. Table 1 demonstrates the convergent of series solution. The impacts of M, A, Nr, Nc, λ and Ha on skin friction $-C_f\, Re_x^{1/2}$ are shown in Table 2. From Table 2 it is noticed that high values of M, A, Nr, Nc, λ increase $-C_f\, Re_x^{1/2}$. The variations in local Nusselt number $g'(0)$ for different parameters Pr, Nb, Nt and Rd have been presented in Table 3. The variations in local motile micro-organism density $\chi'(0)$ for different parameters Γ, Nc, Pr and Pe have been shown in Table 4. It is also noticed that enhancing the values of Γ and Pr local motile organisms' density is rapidly enhanced while enhancing the values of Nc and Pe local motile organisms' density becomes weaker. In Table. 1, the effect is shown with magnetic field Ha=1 for the converges of HAM for the various order of approximation. In Table 2, various values are taken for magnetic field Ha=0.1~1.0 for the physical appearance of effective skin friction for different parameters. There is no fixation of magnetic field Ha.

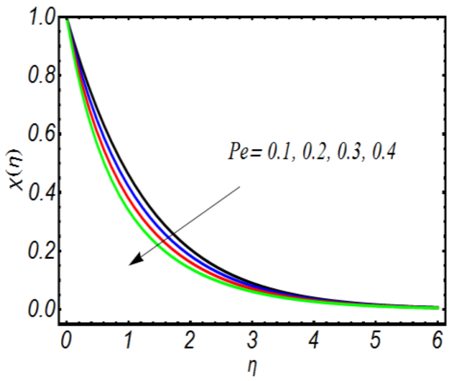

Fig. 19 Impact of Pe on $\chi(\eta)$

6. Conclusions

In this paper the two-dimensional bio-convection flow of Eyring-Powell nanoliquid along convective boundary conditions and variable viscidity over a stretchable surface with motile microorganisms is observed. The influence of MHD, thermal radiation, heat source/sink and activation energy is also investigated. To compute the problem and solution series analytic method is applied and evaluated by using homotopy Analysis approach (HAM). The velocity, temperature, concentration and microorganisms fields are figure out to examine the effect of different parameters. The important results of our analysis are mentioned below:

- By enhancing M (fluid parameter) the values of velocity enhance. On the other hand, the value of velocity is decreased by enhancing variable viscidity parameter A, fluid parameter λ, magnetic field Ha, buoyancy ratio parameter Nr, Rayleigh number Nc and relation of buoyancy force to force of viscidity Γ.
- The temperature of Eyring-Powell nanofluid is increased for Brownian motion Nb, thermophoresis parameter Nt, thermal radiation Rd while reverse behavior is examined for concentration field. In case of heat source the thickness of thermal boundary layer and the temperature increase related to effects of heat sink.
- The concentration of motile microorganism χ has minimum behavior by enhancing the values of Lb and Pe.
- The density of motile micro-organisms is reducing function of bio-convected Lewis number and bio-convection Peclet number.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article

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