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Small scale computational vibration of double-walled CNTs: Estimation of nonlocal shell model

Sehar Asghar[1], Mohamed Amine Khadimallah[*2,3], Muhammad N. Naeem[1], Madiha Ghamkhar[4], Khaled Mohamed Khedher[5,6], Muzamal Hussain[1], Souhail Mohamed Bouzgarrou[7], Zainab Ali[1], Zafar Iqbal[8,9], S.R. Mahmoud[10], Ali Algarni[11], Muhammad Taj[12] and Abdelouahed Tounsi[13,14]

[1]Department of Mathematics, Govt. College University Faisalabad, 38040, Faisalabad, Pakistan
[2]Prince Sattam Bin Abdulaziz University, College of Engineering, Civil Engineering Department, BP 655, Al-Kharj, 11942, Saudi Arabia
[3]Laboratory of Systems and Applied Mechanics, Polytechnic School of Tunisia, University of Carthage, Tunis, Tunisia
[4]Mathematics and Statistics Department, University of Agriculture, Faislabad, Pakistan
[5]Department of Civil Engineering, College of Engineering, King Khalid University, Abha 61421, Saudi Arabia
[6]Department of Civil Engineering, High Institute of Technological Studies, Mrezgua University Campus, Nabeul 8000, Tunisia
[7]Civil Engineering Department, Faculty of Engineering, Jazan University, Kingdom of Saudi Arabia
[8]Department of Mathematics, University of Sargodha, Sargodha, Punjab, Pakistan
[9]Department of Mathematics, University of Mianwali, Punjab, Pakistan
[10]GRC Department, Faculty of Applied studies, King Abdulaziz University, Jeddah, Saudi Arabia
[11]Statistics Department, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
[12]Department of Mathematics, University of Azad Jammu and Kashmir, Muzaffarabad, 1300, Azad Kashmir, Pakistan
[13]Materials and Hydrology Laboratory University of Sidi Bel Abbes, Algeria Faculty of Technology Civil Engineering Department, Algeria
[14]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia

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Abstract. In this paper, vibration characteristics of double-walled carbon nanotubes (CNTs) is studied based upon nonlocal elastic shell theory. The significance of small scale is being perceived by developing nonlocal Love shell model. The wave propagation approach has been utilized to frame the governing equations as eigen value system. The influence of nonlocal parameter subjected to diverse end supports has been overtly analyzed. An appropriate selection of material properties and nonlocal parameter has been considered. The influence of changing mechanical parameter Poisson's ratio has been investigated in detail. The dominance of boundary conditions via nonlocal parameter is shown graphically. The results generated furnish the evidence regarding applicability of nonlocal shell model and also verified by earlier published literature.

Keywords: vibration; nonlocal parameter; Poisson's ratio; double-walled CNTs; Love shell theory

1. Introduction

A diverse range of nano-sized structures are broadly studied to explore vibrational features.

Carbon nanotubes (CNTs) is such discovery by Iijima (1991) that may be used in a variety of fields like material reinforcement, aerospace, medicine, defense and microelectronic devices (Sosa *et al.* 2014, Soldano 2015, Fakhrabadi*et al.* 2015, Moradi *et al.* 2017, Bouadi *et al.* 2018). Owing the striking mechanical properties through the cylindrical mechanism CNTs hold purposeful role in conveying fluid and gas. With a vast area of potential innovation, however CNTs demands more understanding to investigate its mechanical properties. Free vibration analysis of CNTs have been influential aspect in dynamical science for the last one decade. Vibration characteristics are investigated using thin shell theory by Yakobson *et al*. 1996), beam theory by Wang *et al*. (2006) and nonlocal beam theory (Zermi *et al.* 2015, Youcef *et al.* 2018). An eminent study found in based upon ring theory by Vodenitcharova and Zhang (2003) whereas theories of continuum models developed by Li and chou (2003) in literature. Well known two main classes of models used to analyze the theoretical aspects of CNTs have been atomic model and other is continuum model. The classical molecular dynamics (MD) has shown to exceed those of other techniques such as ab initio and tight-binding MD included in class of atomic modeling (Iijima *et al.* 1996, Yakobson *et al.* 1997, Hernandez *et al.* 1998, Sanchez *et al.* 1999, Qian *et al.* 2002). The main reason continuum mechanics (Yoon *et al.* 2003, Fu *et al.* 2006, Ansari *et al.* 2011) became pronounced tool is its computational competence to generate results of large range system in nanometer range. In particular, CNTs have been one of the leading minuscule structure appealed scientists and researchers to analyse experimentally and theoretically its potential aspects. More than a few researchers have been discussed linear and nonlinear vibrational characteristics using Eringen's nonlocal elasticity theory (1983, 2002) by incorporating into beam models. Major focus has been observed on the free vibrational response of CNTs

∗Corresponding author, Assistant Professor
E-mail: am.khadimallah@gmail.com,
mohamedamine.khadimallah@fsgf.rnu.tn

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especially among other nano structures. Additionally, in the age of technology nano sized structures have become utmost importance with large variety of applications in several fields. Since structure of double-walled CNTs consider as two concentric cylindrical shells. Ramteke *et al*. (2019) developed a geometrical model for the analysis and modelling of the uniaxial functionally graded structure using the higher-order displacement kinematics with and without the presence of porosity including the distribution. Additionally, the formulation is capable of modelling three different kinds of grading patterns ie, Power-law, sigmoid and exponential distribution of the individual constituents through the thickness direction. Cylindrical shells are often used to store and transport high-pressure gases and liquids for various hydraulic applications. Pragmatically, their application have been seen in chimney design, pipe flow, aircraft fuselages, ships and construction of buildings. Sophie German was the first who initiated the study of vibration of circular cylindrical shells in 1821. At the end of nineteenth century, Rayleigh (1882) improved the shell analysis. It was Love (1888), who gave the first proper linear shell theory based on Kirchhoff's hypothesis for plates and is also known as the 'first-order approximation shell theory'. His theory has been a basis for developing the modern shell theories by amending physical terms. Kolahchi and Cheraghbak (2017) studied with the nonlocal dynamic buckling analysis of embedded microplates reinforced by single-walled carbon nanotubes (SWCNTs). The material properties of structure are assumed viscoelastic based on Kelvin-Voigt model. Mehar *et al*. (2016) modeled mathematically based on the higher order shell theory. The material properties of carbon nanotube reinforced composite plate are assumed to be temperature dependent and graded in the thickness direction using different grading rules. Kolahchi (2017) investigated the bending, buckling and buckling of embedded nano-sandwich plates based on refined zigzag theory (RZT), sinusoidal shear deformation theory (SSDT), first order shear deformation theory (FSDT) and classical plate theory (CPT). In order to present a realistic model, the material properties of system are assumed viscoelastic using Kelvin-Voigt model. Zhang *et al*. (2001) investigated the free vibrational behaviour of thin cylindrical shells engaging the Love shell equations. They illustrated the comparison for clamped-clamped and simply supported -simply supported end supports by means of wave propagation approach. Finite element method has been used to present thin shell segmentation magnified with cohesive fracture (Cirak *et al.* 2005). Bilouei *et al.* (2016) used as concrete the most usable material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction.

Kolahchi *et al.* (2016a) concerned with the dynamic stability response of an embedded piezoelectric nanoplate made of polyvinylidene fluoride (PVDF). Krichhoff love shell assumptions have been exploited to demonstrate petalling in thin aluminium plates. Bisen *et al*. (2018) and Mehar and Panda (2019) studied the structural response of reinforced material and FG-CNT using the numerical and experimental properties. The results are verified with the open existing literature. The computer software MATLAB was used for the frequency results. The higher order finite element and higher order mid-plane kinematics. The mixture rule was defined for the different materials. Arani and Kolahchi (2016) used a concrete material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. Wang *et al*. (2011) studied the effects of the viscous fluid on the propagation characteristics of elastic waves in carbon nanotubes. Based on the nonlocal continuum theory, the small scales effects are also considered. The equations of wave motion are derived and the dispersion relation is presented. Numerical simulations are performed with the consideration of different scale coefficients to discuss the influence of the viscous fluid. From the results, it can be observed that the dispersion relation can be changed by the fluid viscosity obviously. Zamanian *et al.* (2017) considered the use of nanotechnology materials and applications in the construction industry. However, the nonlinear buckling of an embedded straight concrete columns reinforced with silicon dioxide ($SiO2$) nanoparticles is investigated in the present study. The foundation around the column is simulated with spring and shear layer. Mehar *et al*. (2017a, b) studied the frequency response of FG CNT and reinforced CNT using the simple deformation theory and Mori-Tanaka scheme. They investigated a new frequency phenomena with the combination of Lagrange strain, Green-Lagrange, for double curved and curved panel of FG and reinforced FG CNT. The characteristics of sandwich and grades CNT was found with labeling the temperature environ. The thermoelastic frequency of single shaollow panel was determined using Mori-Tanake formaulation. The research of these authore have opened a new frequency spectra for other material researchers. Kolahchi *et al.* (2017) studied the dynamic buckling of sandwich nano plate (SNP) subjected to harmonic compressive load based on nonlocal elasticity theory. The material properties of each layer of SNP are supposed to be viscoelastic based on Kelvin-Voigt model. In order to mathematical modeling of SNP, a novel formulation, refined Zigzag theory (RZT) is developed. Furthermore, the surrounding elastic medium is simulated by visco-orthotropic Pasternak foundation model in which damping, normal and transverse shear loads are taken into account. Rabczuk *et al*. (2007) exhibited meshfree approach to analyse the capacity of mesh free fluid model with vigorous fracture of fluid filled cylindrical shell that jolted by penetrated projectile. The formulation of fluid shell model was based on Krichhoff Love theory. Motezaker and Eyvazian (2020) deals with the buckling and optimization of a nanocomposite beam. The agglomeration of nanoparticles was assumed by Mori-Tanaka model. Kolahchi and Bidgoli (2016) presented a model for dynamic instability of embedded single-walled carbon nanotubes (SWCNTs). SWCNTs are modeled by the sinusoidal shear deformation beam theory (SSDBT). The modified couple stress theory (MCST) is considered in order to capture the size effects. Malhari Ramteke *et al*. (2020a) obtained the finite element solutions of static deflection and stress value for the functionally graded structure considering variable grading patterns (power-law, sigmoid and exponential) including the porosity effect. The unknown values are obtained computationally via a customized computer code with the help of cubic-order displacement functions considering the varied distribution of porosity (even and uneven) through the panel thickness. Madani *et al.* (2016) presented vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT)- reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions. The structure is subjected to an applied voltage in thickness direction which operates in control of vibration behavior of system. Liew and Wang (2007) discussed wave dispersion for highest modes of single- and double-walled CNTs. At one time thin and thick Love and Cooper Naghdi shell theories have been used to study shear and inertia significance. The feasibility and effective use of elastic models have been explained by comparison of outputs. Kolahchi *et al.* (2016b) investigated the nonlinear dynamic stability analysis of embedded temperature-dependent viscoelastic plates reinforced by single-walled carbon nanotubes (SWCNTs). Motezaker and Kolahchi, (2017a) Investigated the Seismic response of the concrete column covered by nanofiber reinforced polymer (NFRP) layer. Mehar and Panda (2018) investigated the curved shell and CNT vibration with thermal environment using higher order deformation theory. These CNT was mixed with different configurations of the layers. The results have been verified with the earlier investigations. Motezaker and Kolahchi (2017b) presented the dynamic analysis of a concrete pipes armed with Silica ($ SiO_2 $) nanoparticles subjected to earthquake load. Wang *et al*. (2016) investigated the nonlinear vibration of a carbon nanotube which is subjected to the external parametric excitation. By the nonlocal continuum theory and nonlinear von Kármán beam theory, the governing equation of the carbon nanotube is derived with the consideration of the large deformation. The principle parametric resonance of the nanotube is discussed and the approximation explicit solution is presented by the multiple scale method. Numerical calculations are performed. It can be observed that when the mode number is 1, the stable region can be significantly changed by the parametric excitation, length-to-diameter ratio and matrix stiffness. Kolahchi *et al.* (2017) focused with general wave propagation in a piezoelectric sandwich plate. The core is consisted of several viscoelastic nanocomposite layers subjected to magnetic field and is integrated with viscoelastic piezoelectric layers subjected to electric field. The piezoelectric layers play the role of actuator and sensor at the top and bottom of the core, respectively.

Basirjafri *et al*. (2012) obtained the radial breathing mode (RBM) frequency by using thin shell theory in reliance on Hamilton's principle of single-walled CNTs and results included the influence on variation of Poisson's ratio. Malhari Ramteke *et al*. (2020b) studied the two directional graded structure has been developed using a commercial FE package ANSYS and the subsequent deflection responses. Additionally, the model includes the porosity within the graded structure considering even type of distribution pattern. The present model is derived using the basic steps available in the ANSYS platform through the batch input technique. Motezaker *et al.* (2020) presented the present research post-buckling of a cut out plate reinforced through carbon nanotubes (CNTs) resting on an elastic foundation. Alibeigloo and Shaban (2013) inquired the significance of nonlocal parameter by employing three dimension elastic theories with Fourier expansion on vibrations of CNTs. They concluded that by increasing the value of nonlocal parameter, the frequency follows a decreasing pattern.

Pandey *et al*. (2019) predicted the effect of an increasing percentage of nanofiller (glass cenosphere) with Glass/Epoxy hybrid composite curved panels modeled mathematically using the multiscale concept and subsequent numerical eigenvalues of different geometrical configurations (cylindrical, spherical, elliptical, hyperboloid and flat). Wang (2017) explored the nonlinear internal resonance of double-walled nanobeams under the external parametric load. The nonlocal continuum theory is applied to describe the nano scale effects and the nonlinear governing equations are derived by the multiple scale method. The parametric internal resonance is considered and the relation between the frequency and amplitude is discussed. From the numerical simulation, it can be observed that small scale effects are more obvious for short structures. Torkaman *et al*. (2015) conducted the analysis on vibrations and steadiness of rotating single-walled CNTs premised on nonlocal elasticity theory and assumptions considered from Love theory. Exact and authentic results have been established through nonlocal model indicating the influence of rotation rates and role of elasticity for rotating devices. Mehar and Panda (2018) computed the vibration behavior, bending and dynamic response of FG reinforced CNT finite element method. For the sake of generality, the mathematical model was presented with the mixture of Green Lagrange method. The convergence of these methodologies has been checked for the variety of results. The composite plates with different graded was investigated with isotropic and core phase. Hussain and Naeem (2019) studied the vibration features of functionally graded single-walled CNTs based upon modified Love shell theory. The significance of angular velocity and aspect ratios related to length and height on rotating CNTs along with ring support. Galerkin method was employed to formulate governing equations of model and also provided with comparison of rotating and non-rotating frequencies. Mehar *et al*. (2018) evaluated the frequency behavior of nanolpate structure using FEM including the nonlocal theory of elasticity. Computer generated results are created by using the software first time roubustly to check the vibration of nanoplate. The efficiency was checked by comparing the results of available data. Aminikhah and Hemmatnezhad (2011) investigated the homotopy perturbation method to the problem of the nonlinear oscillations of multiwalled carbon nanotubes embedded in an elastic medium under various boundary conditions. Pine *et al*. (2011) investigated the single-walled carbon nanotubes (SWCNTs) have three distinct structures: armchair, zigzag, and chiral. It is known that they have different electronic properties, but the situation regarding their vibrational behavior is less clear.

Several researchers used different approaches for the investigation of frequency of cylinders and concrete material (Kagimoto *et al.* 2015, Mesbah and Benzaid 2017, Alijani and Bidgoli 2018, Demir and Livaoglu 2019, Samadvand and Dehestani 2020) and other other methods as finite element formulation (Dewangan *et al.* 2020, Kunche *et al.* 2019), Grey Wolf algorithm (Kolahchi *et al.* 2020), (Kolahchi *et al.* 2017), and viscoelastic cylindrical shell (Hosseini and Kolahchi 2018, Hajmohammad *et al.* 2018c). The nonlocal elastic shell theory provides a better prediction of the frequency relationships than the beam model and has significant influence on the vibration of DWCNTs for small scale frequencies (Hu *et al.* 2008, Khademolhosseini *et al.* 2009). Moreover, beam model is used wave and vibration properties of the nanotubes based on the presented nonlocal beam equations for scale effects (Lu *et al.* 2007).

In the literature, many researchers used different values of the nonlocal parameter e_0 (Krishnan *et al.* 1998, Wang *et al.* 2007, Hu *et al.* 2008) and here, the authors compared different values of nonlocal parameters to observe the effect on the vibration of DWCNTs. The foremost intension of this paper to investigate vibrations characteristics of double-walled CNTs by means of nonlocal elasticity shell model. The nonlocal shell model is established by inferring the nonlocal elasticity equations in to Love shell theory, which is our particular motivation. The suggested method to investigate the solution of fundamental eigen relations is wave propagation, which is a well-known and efficient technique to develop the fundamental frequency equations. It is carefully observed from the literature, no information is seen regarding present established model where such problem has been considered so it became an incentive to conduct current study. The specific influence of four different end supports based on nonlocal FSM such as clamped-clamped (C-C), clamped-simply supported (C-F) and simply supported-simply supported (S-S) with assorted values of nonlocal parameter are examined in detail.

2. Mathematical formulation

The motion of double-walled CNTs predominates the nonlocal resultant forces and moments so expression for these forces are written as

$$\{N_{xx}, N_{\theta\theta}, N_{x\theta}\} - (e_o a)^2\nabla^2\{N_{xx}, N_{\theta\theta}, N_{x\theta}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}}(\tilde{\sigma}_{xx}, \tilde{\sigma}_{\theta\theta}, \tilde{\sigma}_{x\theta})dz \tag{1}$$

$$\{M_{xx}, M_{\theta\theta}, M_{x\theta}\} - (e_o a)^2\nabla^2\{M_{xx}, M_{\theta\theta}, M_{x\theta}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}}(\tilde{\sigma}_{xx}, \tilde{\sigma}_{\theta\theta}, \tilde{\sigma}_{x\theta})zdz \tag{2}$$

here $\tilde{\sigma}_{xx}$ and $\tilde{\sigma}_{\theta\theta}$stands for stress factors along the axial and tangential directions respectively and $\tilde{\sigma}_{x\theta}$ indicates the shear stress in $x\theta$-plane.

The two-dimensional Hooke's law describes the elements of stress vector in Eqs. (1)-(2) as.

$$\begin{pmatrix}\tilde{\sigma}_{xx}\\ \tilde{\sigma}_{\theta\theta}\\ \tilde{\sigma}_{x\theta}\end{pmatrix} - (e_o a)^2\nabla^2\begin{pmatrix}\tilde{\sigma}_{xx}\\ \tilde{\sigma}_{\theta\theta}\\ \tilde{\sigma}_{x\theta}\end{pmatrix} = \begin{bmatrix}\hat{Q}_{11} & \hat{Q}_{12} & 0\\ \hat{Q}_{12} & \hat{Q}_{22} & 0\\ 0 & 0 & \hat{Q}_{66}\end{bmatrix}\begin{pmatrix}e_{xx}\\ e_{\theta\theta}\\ e_{x\theta}\end{pmatrix} \tag{3}$$

In same manner e_{xx} and $e_{\theta\theta}$ exhibit the strain in x- and θ-directions and $e_{x\theta}$ presents the shear strain in the $x\,\theta$-plane.

For CNTs,$\hat{Q}_{kl}$ $(k, l = 1,2,\ldots 6)$ symbolizes stiffness as functions of Young's modulus and Poisson's ratio written as

$$\hat{Q}_{11} = \frac{E}{1-\nu^2} = \hat{Q}_{22}, \hat{Q}_{12} = \frac{\nu E}{1-\nu^2} = \hat{Q}_{21}, \quad \hat{Q}_{66} = \frac{E}{2(1+\nu)} \tag{4}$$

here E, ν and $\hat{Q}$ are Young's modulus, Poisson's ratio and shear modulus respectively.

Love (1952) submitted the first thin shell theory on base of Kirchhoff's conception. Moreover, an additional modified form of thin shell theory (1963) is established. The diverse range of analytical assessment and comparisons have been examined of these shell theories by Markûs (1988). The components of the strain vector (e) in Eq. (3), that are considered by Love (1952) can be expressed as linear combinations

$$e_{xx} = e_{11} + z\kappa_{11}, e_{\theta\theta} = e_{22} + z\kappa_{22}, \quad e_{x\theta} = e_{12} + 2z\kappa_{12} \tag{5}$$

κ_{11}, κ_{22}, and κ_{12} are known as surface curvatures whereas e_{11} , e_{22} and e_{12} signify the reference surface strains.

Since from Love's theory, the expressions of relation between strain and curvature displacement functions are considered as

$$[e_{11}, e_{22}, e_{12}] = \left[\frac{\partial u}{\partial x}, \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right)\right]$$
$$[\kappa_{11}, \kappa_{22}, \kappa_{12}] = \begin{bmatrix}-\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right),\\ -\frac{2}{R}\left(\frac{\partial^2 w}{\partial x\partial \theta} - \frac{3}{4}\frac{\partial v}{\partial x} + \frac{1}{4R}\frac{\partial u}{\partial \theta}\right)\end{bmatrix} \tag{6}$$

By substituting Eqs. (3) to (6), nonlocal resultant forces and moments takes the following form.

$$N_{xx} - (e_o a)^2\nabla^2 N_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{\sigma}_{xx}dz = \frac{Eh}{1-\nu^2}\frac{\partial u}{\partial x} + \frac{\nu Eh}{1-\nu^2}\frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) \tag{7a}$$

$$N_{\theta\theta} - (e_o a)^2\nabla^2 N_{\theta\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{\sigma}_{\theta\theta}dz = \frac{\nu Eh}{1-\nu^2}\frac{\partial u}{\partial x} + \frac{Eh}{1-\nu^2}\frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) \tag{7b}$$

$$N_{x\theta} - (e_o a)^2\nabla^2 N_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{\sigma}_{x\theta}dz = \frac{Eh}{2(1+\nu)}\left(\frac{1}{R}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial \theta}\right) \tag{7c}$$

$$M_{xx} - (e_o a)^2\nabla^2 M_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{\sigma}_{xx}dz = \frac{\nu D}{R^2}\frac{\partial v}{\partial \theta} - D\left(\frac{\nu}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial x^2}\right) \tag{7d}$$

$$M_{\theta\theta} - (e_o a)^2\nabla^2 M_{\theta\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{\sigma}_{\theta\theta}dz = \frac{D}{R^2}\frac{\partial v}{\partial \theta} - D\left(\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \tag{7e}$$

$$M_{x\theta} - (e_o a)^2 \nabla^2 M_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{x\theta} z dz = D\frac{(1-\nu)}{R}\left(\frac{\partial u}{\partial\theta} + \frac{3}{4}\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial\theta\partial x}\right) \quad (7f)$$

Meanwhile $D = \frac{Eh^3}{12(1-\nu^2)}$ refers as bending rigidity of shell.

The mass density per unit length ρ_t is defined as

$$\rho_t = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz \quad (8)$$

while ρ entitles as mass density.

The fundamental equations from the Love shell theory are considered as

$$\frac{\partial N_{xx}}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta} - \frac{1}{2R^2}\frac{\partial M_{x\theta}}{\partial\theta} = \rho_t\frac{\partial^2 u}{\partial^2 t}$$
$$\frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial\theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial M_{\theta\theta}}{\partial\theta} + \frac{3}{2R}\frac{\partial M_{x\theta}}{\partial x} = \rho_t\frac{\partial^2 v}{\partial^2 t}$$
$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 M_{\theta\theta}}{\partial^2\theta} + \frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial\theta\partial x} - \frac{N_{\theta\theta}}{R} + p = \rho_t\frac{\partial^2 w}{\partial^2 t} \quad (9)$$

Where p expresses the applied pressure on i tube through van der Waals (vdW) interaction forces. The vdW model explains the effects of interlayer relations among the tubes of double-walled CNTs.

By combining Eqs. (7)-(8) into (9), formulated the system of partial differential equations in form of three unknown field variables $u^i, v^i, w^i (i = 1,2)$ for the ith tube of double-walled CNTs.

$$y_{11}^{(1)}u^1 + y_{12}^{(1)}v^1 + y_{13}^{(1)}w^1 = \rho h\left(\ddot{u}^{(1)} - (e_o a)^2(\ddot{u}^{(1)}{}_{xx} + \frac{1}{R_1{}^2}\ddot{u}^{(1)}{}_{xx}\right) \quad (10a)$$

$$y_{21}^{(1)}u^1 + y_{22}^{(1)}v^1 + y_{23}^{(1)}w^1 = \rho h\left(\ddot{v}^{(1)} - (e_o a)^2(\ddot{v}^{(1)}{}_{xx} + \frac{1}{R_1{}^2}\ddot{v}^{(1)}{}_{xx})\right) \quad (10b)$$

$$y_{31}^{(1)}u^1 + y_{32}^{(1)}v^1 + y_{33}^{(1)}w^1 = \rho h\ddot{w}^{(1)} + w^{(1)}\sum_{\substack{j=1\\ j\neq i}}^{2} c_{1j} - \sum_{\substack{j=1\\ j\neq i}}^{2} c_{1j}\, w^{(j)} - (e_o a)^2 \begin{bmatrix} \rho h(\ddot{w}^{(1)}{}_{xx} + \frac{1}{R_1{}^2}\ddot{w}^{(1)}{}_{\theta\theta}) + \\ (\ddot{w}^{(1)}{}_{xx} + \frac{1}{R_1{}^2}\ddot{w}^{(1)}{}_{\theta\theta})\sum_{\substack{j=1\\ j\neq i}}^{2} c_{1j} \\ -\sum_{\substack{j=1\\ j\neq i}}^{2} c_{1j}(\ddot{w}^{(j)}{}_{xx} + \frac{1}{R_1{}^2}\ddot{w}^{(j)}{}_{\theta\theta}) \end{bmatrix} \quad (10c)$$

$$y_{11}^{(2)}u^2 + y_{12}^{(2)}v^2 + y_{13}^{(2)}w^2 = \rho h\left(\ddot{u}^{(2)} - (e_o a)^2(\ddot{u}^{(2)}{}_{xx} + \frac{1}{R_2{}^2}\ddot{u}_{\theta\theta}{}^{(2)})\right) \quad (10d)$$

$$y_{21}^{(2)}u^2 + y_{22}^{(2)}v^2 + y_{23}^{(2)}w^2 = \rho h\left(\ddot{v}^{(2)} - (e_o a)^2(\ddot{v}^{(2)}{}_{xx} + \frac{1}{R_2{}^2}\ddot{v}^{(2)}{}_{xx})\right) \quad (10e)$$

$$y_{31}^{(2)}u^2 + y_{32}^{(2)}v^2 + y_{33}^{(2)}w^2 = \rho h\ddot{w}^{(2)} + w^{(2)}\sum_{\substack{j=1\\ j\neq 2}}^{2} c_{2j} - \sum_{\substack{j=1\\ j\neq 2}}^{2} c_{2j}\, w^{(j)} - (e_o a)^2 \begin{bmatrix} \rho h(\ddot{w}^{(2)}{}_{xx} + \frac{1}{R_2{}^2}\ddot{w}^{(2)}{}_{\theta\theta}) + (\ddot{w}^{(2)}{}_{xx} \\ + \frac{1}{R_2{}^2}\ddot{w}^{(2)}{}_{\theta\theta})\sum_{\substack{j=1\\ j\neq 2}}^{2} c_{2j} - \sum_{\substack{j=1\\ j\neq 2}}^{2} c_{2j}(\ddot{w}^{(j)}{}_{xx} + \frac{1}{R_2{}^2}\ddot{w}^{(j)}{}_{\theta\theta}) \end{bmatrix} \quad (10f)$$

here $y_{pq} = (p, q = 1,2,3)$ are stated as partial operators can be seen in Appendix.

During the past few years, numerous theories have been extensively debated for vibration of nanotube, shell and plate morphologies of several conformations depending upon certain edge conditions. Wave propagation approach is among the most significant and successfully used numerical technique by researchers to investigate the free vibrations of cylinder-shaped shell, plates and nanotubes. The three modal displacement functions of the shell for *i*th tube can be written as

$$u^{(i)}(x,\theta,t) = a_m \cos(n\theta) e^{(i\omega t - ik_m x)} \quad (11a)$$

$$v^{(i)}(x,\theta,t) = b_m \sin(n\theta) e^{(i\omega t - ik_m x)} \quad (11b)$$

$$w^{(i)}(x,\theta,t) = c_m \cos(n\theta) e^{(i\omega t - ik_m x)} \quad (11c)$$

where a_m, b_m, c_m describe the displacement amplitude coefficients in x, θ and z directions correspondingly. The angular frequency is designated as ω, circumferential wave number by n and k_m referred to be axial wave number allied with end supports obligatory on double-walled CNTs. Replacing the functions and derivatives into the system of fundamental equations, henceforth derived a set of simultaneous as follows

$$Y_{11}{}^{(i)}a_m^i + Y_{12}{}^{(i)}b_m^i + Y_{13}{}^{(i)}c_m^i = -\omega^2(1 - (e_o a)^2\nabla^2)\rho h a_m^i \quad (12a)$$

$$Y_{21}{}^{(i)}a_m^i + Y_{22}{}^{(i)}b_m^i + Y_{23}{}^{(i)}c_m^i = -\omega^2(1 - (e_o a)\nabla^2)\rho h b_m^i \quad (12b)$$

$$Y_{31}{}^{(i)}a_m^i + Y_{32}{}^{(i)}b_m^i + Y_{33}{}^{(i)}c_m^i + (1 - (e_o a)^2\nabla^2)\left[\sum_{\substack{j=1\\ j\neq i}}^{2} c_{ij}c_m^i - \sum_{\substack{j=1\\ j\neq i}}^{2} c_{ij}c_m^i\right] = -\omega^2(1 - (e_o a)^2\nabla^2)\rho h c_m^i \quad (12c)$$

Since $i = (1,2)$ and the algebraic operators $Y_{pq}^{(i)}$ are acquired using Appendix with $p, q = (1,2,3)$.

3. Result and discussion

An inventive approach to fabricate nonlocal Love shell model based on wave propagation technique of double-walled CNTs for vibrational response has been demonstrated and verified with results presented in the literature. Considering the negligible percentage of error, thus it confirms the validation of suggested nonlocal shell model. An innovational nonlocal model to examine the scale effect on vibrational behavior of armchair, zigzag and chiral of double-walled CNTs. The influence of nonlocal parameter with variation of Poisson's ratio are investigated depending upon certain edge supports. The mass density is assumed to be 2300 kg/m^3, with Young's Modulus 1Tpa (Basirjafri *et al.* 2012). The computations of our newly model with proposed approach with same data sets, our results are consistent with previous reports in MD (Zhang *et al.* 2009) for CNTs as shown in Table 1.

In accordance of theoretical procedure, at first frequency of double-walled CNTs with mutation in values of Poisson's ratio is observed as shown in Fig. 1. As figure

Table 1 Comparison of present result with MD simulation (Zhang *et al.* 2009)

L/d	*f* (THz)	
	Present	MD Simulation (Zhang *et al.* 2009)
4.68	1.23445	1.17638
6.67	0.67832	0.56835
8.47	0.44146	0.37294
10.26	0.30922	0.27354
13.89	0.17360	0.12031

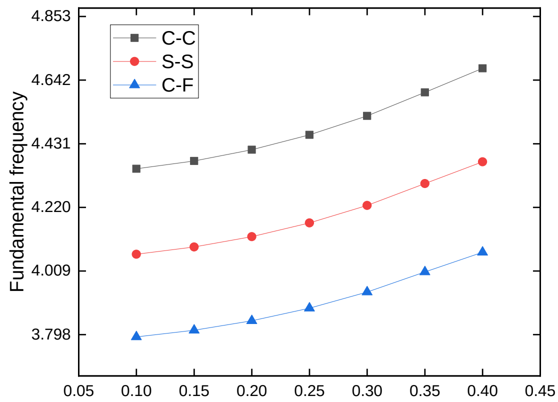

Fig. 1 Frequency variation of arm chair (7, 7) with e_o=0.2 versus Poisson's ratio

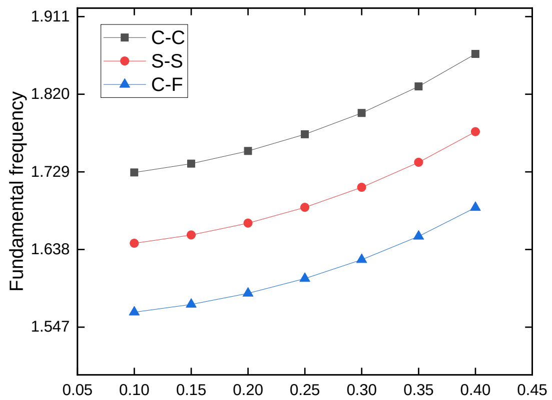

Fig. 2 Frequency variation of arm chair (7, 7) with e_o=0.65 versus Poisson's ratio

shows fundamental frequency rises by rising the Poisson's ratio. The results presented here are in good accordance with those established by Basirjafri *et al.* (2012). The fundamental frequencies are calculated of double-walled CNTs subjected to three distinct end supports C-C, S-S and C-F. The frequencies are obtained for the varying values of Poisson's ratio from 0.1 to 0.4. To inspect significance of nonlocal parameter on vibration of double-walled CNTs, two peculiar values of nonlocal parameter $e_o = 0.2, 0.65$ and 1.2 are considered. When Poisson's ratio varies from $\nu = 0.1$ to 0.4, the frequencies of C-C armchair (7, 7) with $r_1 = 1.5$ against $e_o = 0.2$ are 4.3479, 4.3738, 4.4110, 4.4604, 4.5233, 4.6015 and 4.6811 and S-S frequencies are 4.0646, 4.0886, 4.1229, 4.1685, 4.2266, 4.2989 and 4.3711 respectively. Similarly C-F frequencies are 3.7905, 3.8125, 3.8440, 3.8859, 3.9394,

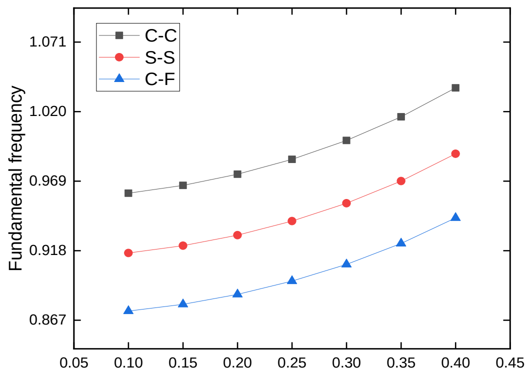

Fig. 3 Frequency variation of arm chair (7, 7) with e_o=1.2 versus Poisson's ratio

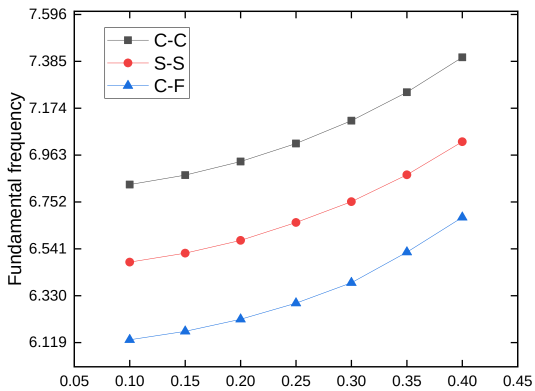

Fig. 4 Frequency variation of zigzag (12, 0) with e_o=0.2 versus Poisson's ratio

4.0058 and 4.0071. The corresponding frequencies have been sketched in Fig. 1. It is noticed that clamped -clamped frequencies are higher followed by simply supported - simply supported and clamped-free and gap between frequency curves are evident. As ratio increases, the frequencies also show gradually slow increasing pattern. Fig. 2 display the frequency curves of armchair (7, 7) against e_0=0.65 subjected to aforementioned end supports. The curves affirm the gap between the end supports as shown in Fig. 1. For the first value of Poisson's ratio ν=0.1 C-C armchair (7, 7) frequency is observed as 1.7283, S-S as 1.6456 and C-F 1.5645 receptively. Similarly at the last value ν=0.4, C-C frequency is 1.8672, S-S and C-F are 1.7761, 1.6871. It is noticed that frequencies decline for all end supports. Furthermore, one important observation is clearly seen that with increased nonlocal parameter value, the frequency tends to decrease for all end conditions. The frequency variation with ratio of armchair (7, 7) double-walled CNTs against e_0=1.2 can be viewed in Fig. 3. The clamped-clamped frequencies are 0.9602, 0.9659, 0.9741, 0.9850, 0.9989, 1.0162 and 1.0374 whereas, clamped-free are calculated as 0.8737, 0.8787, 0.8860, 0.9080, 0.9233 and 0.9421. It is observed once again that frequencies decrease as nonlocal parameter increases. The frequency curve maintains the regular gap between end supports as seen before. It is also concluded that frequencies for these certain values of nonlocal parameter against range of Poisson ratio rise slowly with same parameters and length

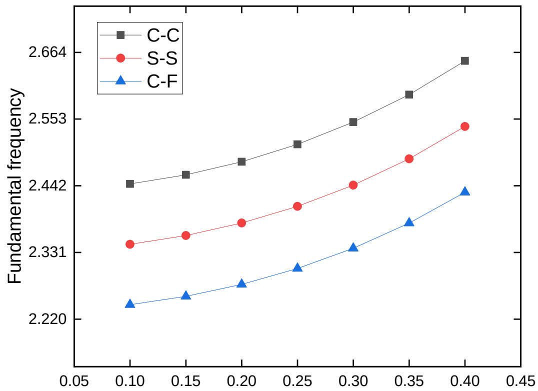

Fig. 5 Frequency variation of zigzag (12, 0) with e_o=0.2 versus Poisson's ratio

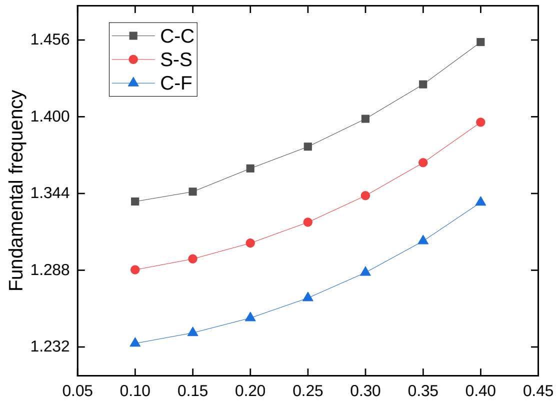

Fig. 6 Frequency variation of zigzag (12, 0) with e_o=0.2 versus Poisson's ratio

double-walled CNTs. Figs. 4-6 illustrate the significance of scale effect on vibration of zigzag (12, 0) double-walled CNTs against Poisson ratio. The scale effect frequency curves are portrayed with e_0=0.2, 0.65 and 1.2 depending upon three boundary conditions such like arm chair (7,7). The C-C frequencies of (12, 0) at e_0=0.2 versus Poisson ratio are 6.8305, 6.8730, 6.9340, 7.0150, 7.1180, 7.2460 and 7.4030, at e_0=0.65 are 2.4450, 2.4602, 2.4820, 2.5110, 2.5479, 2.5937 and 2.6499 and in similar way at e_0=1.2 are 1.3420, 1.3550, 1.3623, 1.3782, 1.3985, 1.4236 and 1.4545 respectively. Over again the trend presents a slow increase as ratio increases but at same time with increased nonlocal parameter, frequencies decrease for all boundary conditions. Another observation is clearly seen from these curves that zigzag possess higher frequencies as compared to armchair double-walled CNTs. It is explained as from basic carbon morphology zigzag owns numerous elements parallel to tube axis but armchair does not possess such feature. Consequently, zigzag CNTs are supposed to show strong bending and longitudinal permanence in comparison to armchair CNTs.

Here in chiral case, Figs. 7-9 elucidate the influence of nonlocal parameters on the frequencies of chiral with indices (8, 3) based on nonlocal love shell model. These frequency curves confirmed the obvious higher values of C-C than those of S-S and C-F chiral double-walled CNTs. When e_0=0.2 then the C-C chiral frequency peaks against

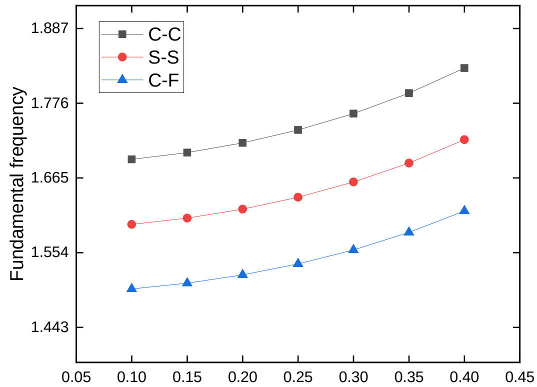

Fig. 7 Frequency variation of chiral (8, 3) with e_o=0.2 versus Poisson's ratio

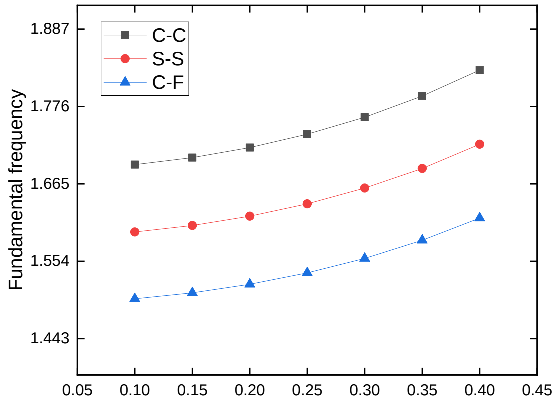

Fig. 8 Frequency variation of chiral (8, 3) with e_o=0.2 versus Poisson's ratio

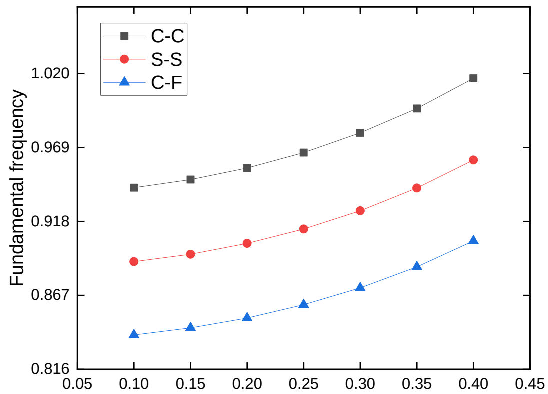

Fig. 9 Frequency variation of chiral (8, 3) with e_o=0.2 versus Poisson's ratio

ratio variation are 4.2260, 4.2510, 4.2870, 4.3347, 4.3955, 4.4713 and 4.5643, for S-S 3.8971, 3.9198, 3.9524, 3.9957, 4.0509, 4.1196, 4.2041 are noticed as and C-F are 3.5737, 3.5941, 3.6233, 3.6621, 3.7116, 3.7734 and 3.8494 drawn for chiral (8, 3) with e_0=0.65 C-C are 1.6926, 1.7027, 1.7170, 1.7362, 1.7605, 1.7909 and 1.8281, S-S are 1.5960, 1.6053, 1.6187, 1.6364, 1.6590, 1.6871 and 1.7218 and C-F are 1.5002, 1.5088, 1.5210, 1.5373, 1.5581, 1.5841 and 1.6159 frequencies are displayed. It can be seen that frequencies are reducing as nonlocal parameter increasing. At the very moment, with higher Poisson's ratio it is noted that frequencies incline as well with respect to all considered end conditions.

4. Conclusions

The frequency patterns have been discussed for armchair, zigzag and chiral double-walled CNTs to inspect the influence of Poisson's ratio based on nonlocal Love shell model. Natural frequency curves are presented for three specific end supports considering distinct values of nonlocal parameter. The fundamental frequency patterns for armchair, zigzag and chiral double-walled CNTs exhibit the resembling trends for varying values of ratio. Although we observe the phenomena for structural strength of double-walled CNTs by comparing the respective frequencies for all types. Because of the fact that greater the Poisson ratio, softer the material, zigzag CNTs possess the larger values than those of arm chair and chiral tubes. Further for all same parameters when we compare the frequencies for chiral and armchair tubes, it is revealed that chiral tubes obtain less frequencies. The chiral frequencies subjected to all nonlocal parameters exhibit lesser values but close to armchair frequencies. For that reason, chiral tubes hold more sustainable structural characteristics as compared to armchair and zigzag tubes. A slow increase in frequencies against variation of Poisson's ratio also indicates insensitivity of it for suggested nonlocal model. In addition, decrease in frequencies with increase in nonlocal parameter authenticates the applicability of nonlocal Love shell model.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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CC

Appendix-I

$$y_{11}{}^{(i)} = \frac{Eh}{1-\nu^2}\frac{\partial^2}{\partial x^2} + \left(\frac{1}{R_i{}^2}\frac{Eh}{2(1+\nu)} - \frac{1}{R_i{}^4}\frac{D(1-\nu)}{8}\right)\frac{\partial^2}{\partial \theta^2}$$

$$y_{12}{}^{(i)} = \frac{1}{R_i}\left(\frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)} - \frac{3}{8}\frac{D(1-\nu)}{R_i{}^2}\right)\frac{\partial^2}{\partial\theta\partial x}$$

$$y_{13}{}^{(i)} = \frac{1}{R_i}\frac{Eh\nu}{1-\nu^2}\frac{\partial}{\partial x} + \left(\frac{D(1-\nu)}{2R_i{}^3}\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial\theta}$$

$$y_{21}{}^{(i)} = y_{12}{}^{(i)}$$

$$g_{22}{}^{(i)} = \left(\frac{Eh}{2(1+\nu)} + \frac{9D(1-\nu)}{8R_i{}^2}\right)\frac{\partial^2}{\partial x^2} + \frac{1}{R_i{}^2}\left(\frac{Eh}{(1-\nu^2)} + \frac{D}{R_i{}^2}\right)\frac{\partial^2}{\partial\theta^2}$$

$$y_{23}{}^{(i)} = -\left(\frac{\nu D}{R_i{}^2} + \frac{3}{2}\frac{D(1-\nu)}{R_i{}^2}\right)\frac{\partial^3}{\partial x^2\partial\theta} - \frac{D}{R_i{}^4}\frac{\partial^3}{\partial\theta^3} + \frac{1}{R_i{}^2}\frac{Eh}{1-\nu^2}\frac{\partial}{\partial\theta}$$

$$y_{31}{}^{(i)} = -y_{13}{}^{(i)}$$

$$y_{32}{}^{(i)} = \left(\frac{D}{R_i{}^2}(2-\nu) + \frac{3D(1-\nu)}{4R_i{}^2}\right)\frac{\partial^3}{\partial x^2\partial\theta} + \frac{D}{R_i{}^4}\frac{\partial^3}{\partial\theta^3}$$

$$-\frac{1}{R_i{}^2}\frac{E\hbar}{1-\nu^2}\frac{\partial}{\partial\theta} y_{33}{}^{(i)} =$$

$$-D\frac{\partial^4}{\partial x^4} - \left(\frac{2D}{R_i{}^2} + \frac{2D(1-\nu)}{R_i{}^2}\right)\frac{\partial^4}{\partial x^2\partial\theta^2} - \frac{D}{R_i{}^4}\frac{\partial^4}{\partial\theta^4} - \frac{1}{R_i{}^2}\frac{E\hbar}{1-\nu^2}$$

Appendix-II

$$Y_{11}{}^{(i)} = \frac{Eh}{1-\nu^2}(-k_m{}^2) + \left(\frac{1}{R_i{}^2}\frac{Eh}{2(1+\nu)} - \frac{1}{R_i{}^4}\frac{D(1-\nu)}{8}\right)(-n^2)$$

$$Y_{12}{}^{(i)} = \frac{1}{R_i}\left(\frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)} - \frac{3}{8}\frac{D(1-\nu)}{R_i{}^2}\right)(-nik_m)$$

$$Y_{13}{}^{(i)} = \frac{1}{R_i}\frac{Eh\nu}{1-\nu^2}(-nik_m) + \left(\frac{D(1-\nu)}{2R_i{}^3}\frac{\partial^2}{\partial x^2}\right)(-n)$$

$$Y_{21}{}^{(i)} = Y_{12}{}^{(i)} Y_{22}{}^{(i)} = \left(\frac{Eh}{2(1+\nu)} + \frac{9D(1-\nu)}{8R_i{}^2}\right)(-k_m{}^2)$$

$$+\frac{1}{R_i{}^2}\left(\frac{Eh}{(1-\nu^2)} + \frac{D}{R_i{}^2}\right)(-n^2)$$

$$Y_{23}{}^{(i)} = \left(\frac{\nu D}{R_i{}^2} + \frac{3}{2}\frac{D(1-\nu)}{R_i{}^2}\right)(nk_m{}^2) - \frac{D}{R_i{}^4}n^3 + \frac{1}{R_i{}^2}\frac{Eh}{1-\nu^2}(-n)$$

$$Y_{31}{}^{(i)} = -Y_{13}{}^{(i)}$$

$$Y_{32}{}^{(i)} = \left(\frac{D}{R_i{}^2}(2-\nu) + \frac{3D(1-\nu)}{4R_i{}^2}\right)(-nk_m{}^2)$$

$$+\frac{D}{R_i{}^4}n^3 - \frac{1}{R_i{}^2}\frac{E\hbar}{1-\nu^2}n$$

$$Y_{33}{}^{(i)} = -Dk_m{}^4 - \left(\frac{2D}{R_i{}^2} + \frac{2D(1-\nu)}{R_i{}^2}\right)n^2k_m{}^2$$

$$-\frac{D}{R_i{}^4}n^4 - \frac{1}{R_i{}^2}\frac{E\hbar}{1-\nu^2}$$