Advances in Concrete Construction, *Vol. 1, No. 2 (2013) 163-185*
DOI: http://dx.doi.org/10.12989/acc.2013.1.2.163

A unified design procedure for preloaded rectangular RC columns strengthened with post-compressed plates

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(Received October 19, 2012, Revised January 23, 2013, Accepted June 26, 2013)

Abstract. The use of post-compressed plates (PCP) to strengthen preloaded reinforced concrete (RC) columns is an innovative approach for alleviating the effects of stress-lagging between the original column and the additional steel plates. Experimental and theoretical studies on PCP-strengthened RC columns have been presented in our companion papers. The results have demonstrated the effectiveness of this technique for improving the strength, deformability and ductility of preloaded RC columns when subjected to axial or eccentric compression loading. An original and comprehensive design procedure is presented in this paper to aid engineers in designing this new type of PCP-strengthened RC column and to ensure proper strengthening details for desirable performance. The proposed design procedure consists of five parts: (1) the estimation of the ultimate load capacity of the strengthened column, (2) the design of the initial pre-camber displacement of the steel plate, (3) the design of the vertical spacing of the bolts, (4) the design of the bearing ends of the steel plates, and (5) the calculation of the tightening force of the bolts. A worked example of the design of a PCP-strengthened RC column is shown to demonstrate the application of the proposed design procedure.

Keywords: reinforced concrete column; pre-cambered steel plate; axial load capacity; design procedure; worked example

1. Introduction

RC columns are often the most critical load-bearing structural members in multi-story concrete buildings. These columns may require strengthening due to having higher loads than those foreseen in the initial design of the structure, material deterioration or fire damage. Steel jacketing has been widely adopted to strengthen RC columns in recent years. Extensive experimental and the oretical studies on steel jacketing (Frangou *et al*. 1995, Cirtek 2001a, b, Giménez *et al*. 2009a, b, Adam *et al*. 2007, 2008a, b, 2009a, b, Calderon *et al*. 2009,Garzón-Roca *et al*. 2011a, b, Yang *et al*. 2007, Li *et al*. 2009,Montuori and Piluso 2009) have been conducted. However, only a few studies have focused on the effects of pre-existing loads on stress-lagging between the concrete core and the new jackets. Ersoy *et al*. (1993), Takeuti *et al*. (2008) and Giménez *et al.* (2009b)

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http://www.techno-press.org/?journal=acc&subpage=7 ISSN: 2287-5301 (Print), 2287-531X (Online)

experimentally investigated the effects of pre-existing loads on axial strengthening efficiency. Their results demonstrated that stress-lagging effects can significantly decrease the ultimate axial load capacity of the strengthened columns. To address the stress-lagging problem generated by pre-existing loads in the concrete, Su and Wang (2012) and Wang and Su (2012a, b) proposed post-compressed plates (PCP) to strengthen preloaded RC columns. The aim of this approach is to alleviate the stress-lagging and displacement incompatibility problems between steel plates and RC columns. The plates provided to strengthen RC columns are slightly longer than the clear height of the column. After fixing the two ends of the plates to the column, the initial form of the plates is slightly cambered. Post-compression of the plates and de-compression of the RC column can be achieved by progressive flattening the plates which can generate a thrust to the column by means of the arching action. Hence, the pre-existing loading can be evenly shared with the RC column and steel plates. The strain difference between the steel plates and concrete can be minimized by controlling the initial camber of the steel plates. In total, eight and nineteen specimens were tested under axial and eccentric compression loading, respectively. The results showed that PCP could actively share the existing axial loads with the original column. Stress relief in the original concrete column and post-stress developed in the steel plates can alleviate the stress-lagging and displacement incompatibility problems. Furthermore, the experiments demonstrated the excellent ultimate load capacity, enhanced deformability and sufficient ductility of PCP-strengthened columns.

Based on the authors' previous experimental and theoretical studies, this paper attempts to establish a unified design procedure to aid engineers in the design of PCP-strengthened columns subjected to static loads. A worked example of the design of a PCP-strengthened column is provided to demonstrate the application of the proposed design procedure.

2. Coupled finite element / reproducing kernel approximation

The ultimate axial load capacity of the PCP-strengthened RC column is controlled by the strength of the steel plates, the initial pre-camber, the strength of the vertical reinforcement, the concrete grade and the vertical spacing of the bolts. Because the concrete grade and strength of the vertical reinforcement of the existing columns cannot be modified, the ultimate axial load capacity of the PCP-strengthened RC column is dictated by the size and strength of the steel plates. The design method is divided into two types according to the applied axial load eccentricity, which is less than or larger than that of the balance failure.

2.1 Estimating the load eccentricity of the RC column

Before choosing the type of strengthening method, the axial load eccentricity of an RC column corresponding to the balance failure should be calculated. The balance failure is referred to the case where concrete strain attains its ultimate value while tension steel bars are yielded. The depth of the compression zone (c_b) of the concrete at a balanced failure is

$$c_b = \frac{d\varepsilon_{cu}E_s}{f_{sy} + \varepsilon_{cu}E_s} \tag{1}$$

where f_{sy} (unit: MPa) and E_s are the yield strength and Young's modulus of the steel bars, respectively, d is the depth of the tension steel measured from the extreme compression fiber,

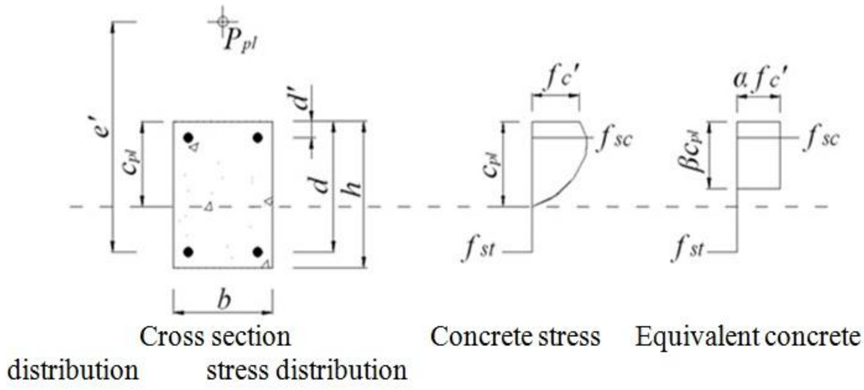
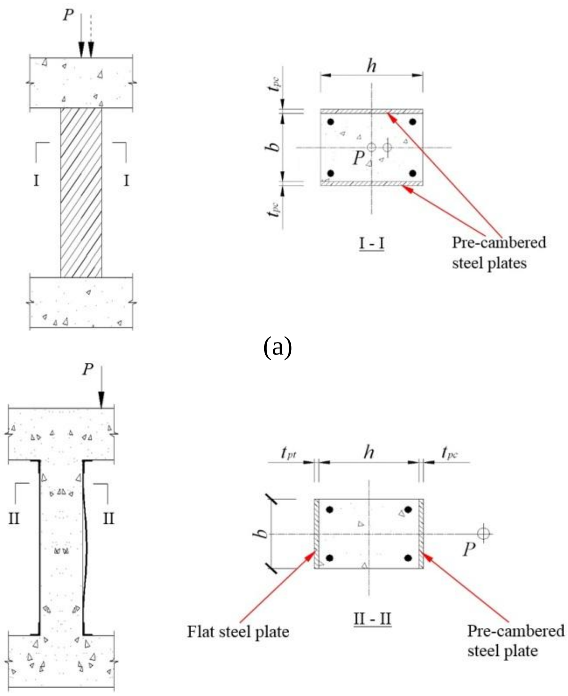


Fig. 1 Concrete stress-block factors



(a)

(b)

Fig. 2 The two arrangements of steel plates: (a) Case 1 and (b) Case 2

and ε_{cu} is the extreme fiber compression strain of the concrete.

By substituting the value of c_b into Eqs. (2) and (3), the compression loading (P_b) and the load eccentricity (e_b) at a balanced failure can be calculated

$$P_b = \alpha\beta c_b b f_c{}' + A_{sc} f_{scy} - A_{st} f_{sct} \tag{2}$$

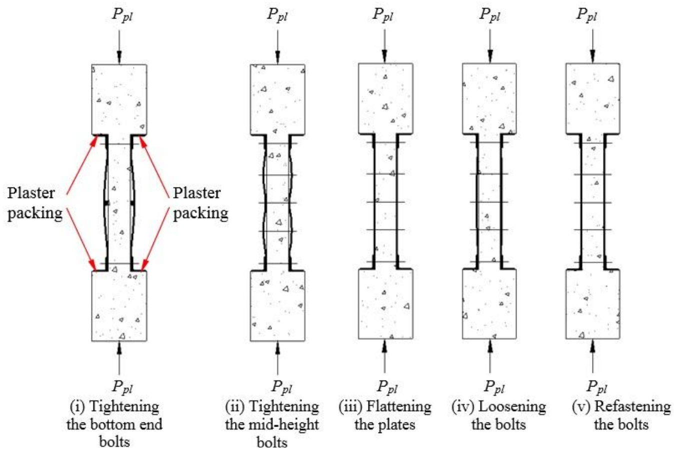
$$P_b\,(e_b + \frac{h}{2} - d') = \alpha\beta c_b b f_c'(d - \frac{\beta c_b}{2}) + A_{sc} f_{scy}(d - d') \tag{3}$$

where α and β are the stress block factors of concrete stress distribution, as shown in Fig. 1, which can be calculated using Eqs. (4) and (5) (Collins and Mitchell 1991)

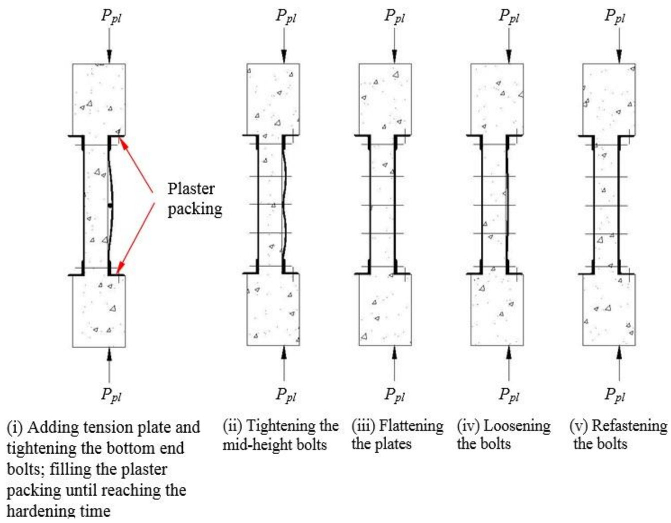
$$\alpha = [\frac{\varepsilon_{cu}}{\varepsilon_{c0}} - \frac{1}{3}(\frac{\varepsilon_{cu}}{\varepsilon_{c0}})^2]/\beta \tag{4}$$

$$\beta = (4 - \frac{\varepsilon_{cu}}{\varepsilon_{c0}})/(6 - \frac{2\varepsilon_{cu}}{\varepsilon_{c0}}) \tag{5}$$

where ε_{co} is the concrete compressive strain corresponding to the concrete cylinder compressive strength f_c' (unit: MPa).



(a)



(b)

Fig. 3 Post-stressed procedure: (a) Case 1 and (b) Case 2

2.2 Calculation of the depth of the compression zone (c_{pl}) and strain of concrete ($\varepsilon_{c,pl}$) in the preloading stage

In the preloading stage, the vertical force equilibrium Eq. (6) of the RC column can be obtained from the sum of the internal forces.

$$P_{pl}' = \alpha\beta b c_{pl} f_c' \left(\frac{2\varepsilon_{c,pl}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,pl}^2}{\varepsilon_{c0}}\right) + E_{sc}A_{sc}\varepsilon_{c,pl}\left(\frac{c_{pl}-d'}{c_{pl}}\right) - E_{st}A_{st}\varepsilon_{c,pl}\left(\frac{d - c_{pl}}{c_{pl}}\right) \tag{6}$$

where P_{pl}' is the applied vertical load in the working condition.

The equation obtained from taking moments about the tension steel is

$$P_{pl}'e' = \alpha\beta b c_{pl} f_c' \left(\frac{2\varepsilon_{c,pl}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,pl}^2}{\varepsilon_{c0}}\right)\left(d - \frac{\beta c_{pl}}{2}\right) + E_{sc}A_{sc}\varepsilon_{c,pl}\left(\frac{c_{pl} - d'}{c_{pl}}\right)(d - d') \tag{7}$$

where d and d' are the depths of the tension steel and the compression steel measured from the extreme compression fiber, respectively; E_{sc} and E_{st} are Young's modulus of the compression steel bar and the tension steel bar, respectively, and A_{sc} and A_{st} are the total cross-sectional areas of the compression steel bars and the tension steel bars, respectively.

The depth of the compression zone (c_{pl}) and the strain of concrete ($\varepsilon_{c,pl}$) in the preloading stage can be obtained from Eqs. (4) - (7).

2.3 Twotypes of strengthening methods

After obtaining the load eccentricity of the RC column corresponding to the balance failure, a suitable type of strengthening method can be determined.

Type 1. $0 \le e \le e_b$

The preloaded RC column is strengthened with PCP under pure axial or small eccentric compression loading. Fig. 2(a) shows the arrangement of the side plates, which are parallel to the load eccentricity direction. Such an arrangement can maximize the axial load resistance with a minor increase in the flexural capacity.

The corresponding post-stressed procedure, developed by Su and Wang (2012) and Wang and Su (2012a, b), is introduced in Fig. 3(a). The process can be divided into the following steps: (i) the bolts at both ends of the columns are tightened, and the gaps between the concrete and the steel plates are filled with plaster until hardened; (ii) the bolts at mid-height are tightened, andthe buckling mode of the pre-cambered plate is thus changed to higher modes; and (iii) the plates are flattened by tightening the rest of the bolts (iv) to achieve a more evenly distributed internal stress in the plates. Then, all of the bolts are slightly loosened (v) and refastened.

Type 2. $e > e_b$

The preloaded RC columns are strengthened with PCP under large eccentric compression loading. In this type of strengthening method, the flat and pre-cambered steel plates are placed on the tension and compression sides of the RC column, respectively, as shown in Fig. 2(b). The decompression method is only applied to the compression side. This plate arrangement can effectively increase the flexural capacity of the strengthened column.

The corresponding post-stress procedure is presented in Fig. 3(b). The process can be divided into the following steps: (i) the tensioned steel plate and bolts, which are installed at both ends of the columns are tightened, and the gaps between the steel angles and the concrete at the base and top of the steel plates are filled with an injection plaster to form a layer of bedding between the steel angles and the concrete; (ii) the bolts at mid-height are tightened until reaching the hardening time of the plaster; thus, the buckling mode of the pre-cambered plate is changed to a higher mode; (iii) the plates are flattened by tightening the rest of the bolts (iv) to achieve a more evenly distributed internal stress in the plates. Then, all of the bolts are slightly loosened (v) and refastened.

2.4 Steel plate installed on side faces of the RC column

In practice, the RC columns to be strengthened are usually subjected to eccentric pre-compressed axial loads. According to the experimental and theoretical results obtained fromSu and Wang (2012) and Wang and Su (2012a, b), placing pre-cambered steel plates on the side faces of an RC column and applying a decompression method to both of them can significantly improve the strength, deformability and ductility of columns under a pure axial load ($e = 0$) or small eccentric compression loads ($0 < e \le e_b$). In this section, a comprehensive design procedure will be presented. The relationship between the initial pre-camber and the plate thickness in the ultimate load stage is shown.

2.4.1 Determination of the required load capacity of the strengthened column in the working condition

Step 1. The original ultimate load capacity of the un-strengthened column

According to ACI 318-02 and AISC 360-05, the strength reduction factor of bearing on concrete is φ_b; the strength reduction factor of shear section on concrete is φ_s; the strength reduction factors of the tension-controlled and compression-controlled sections are φ_t and φ_c, respectively. For the steel plate, the safety factors for compression and tension are χ_c and χ_t, respectively. The partial safety factor for dead loads is γ_G.

The vertical force equilibrium Equation of the RC column can be obtained from the sum of the internal forces

$$\gamma_G P_{ori}' \le \varphi_c(\alpha\beta b c_{ori} f_c' + A_{sc} f_{scy} - A_{st} f_{st}) \tag{8}$$

where P_{ori}' is the required load capacity of the un-strengthened column in the working condition and c_{ori} is the depth of the compression zone. The subscript *ori* represents the original state of the RC column.

The moment equilibrium Eq., which was obtained by taking moments about the tension steel, is given by

$$\gamma_G P_{ori}' e' \le \varphi_c \left[\alpha\beta b c_{ori} f_c' \left(d - \frac{\beta c_{ori}}{2} \right) + A_{sc} f_{scy}(d - d')\right] \tag{9}$$

where e' is the distance between the loading point and the tension steel bars. If the RC column is subjected to axial compression loading, the e' and A_{st} values are equal to zero and f_{scy} is the yield strength of the compressive steel bars (unit: MPa). The depth of the compression zone (c_{ori}) and the required load capacity of the un-strengthened column in the working condition (P_{ori}') can be determined from Eqs. (4), (5), (8) and (9).

Step 2. Determining the size of the steel plates

The required load capacity of the strengthened column in the working condition (P_{pre}') is given by

$$\gamma_G P_{pre}^{'} \leq \gamma_G P_{ori}^{'} + 2t_{pc}b_{pc}E_{pc}\varepsilon_{pc}\chi_c \tag{10}$$

where b_{pc} is the width of the steel plate, and ε_{pc} is the strain of the steel plate at the ultimate load stage, which can be written as

$$\varepsilon_{pc} \geq \frac{\gamma_G(P_{pre}^{'}-P_{ori}^{'})}{2A_{pc}\, E_{pc}\, \chi_c} \tag{11}$$

For an optimal design, the steel plates should reach their ultimate load capacity prior to the occurrence of the compressive failure of concrete. Hence, the steel plate strain at the ultimate loading stage ε_{pcy} should be equal to or greater than ε_{pc}. The size of steel plates can be determined by setting ε_{pcy} equal to ε_{pc}.

2.4.2 Determination of the initial pre-camber of the steel plates

Step 1. Calculation of the strains of concrete ($\varepsilon_{c,ps}$)and the steel plate($\varepsilon_{pc,ps}$)in the post-stressing stage

To make steel plates to reach their ultimate load capacity prior to the occurrence of the compressive failure of concrete, after obtaining the strain value of the steel plate (ε_{pc}) at the ultimate load, the concrete strain ($\varepsilon_{c,ps}$) at the post stressing stage is expressed as

$$\varepsilon_{c,ps} \leq \varepsilon_{cu} - \varepsilon_{pc} \tag{12}$$

The relationship between the strain of the concrete ($\varepsilon_{c,ps}$) and the strain of the steel plate ($\varepsilon_{pc,ps}$) in the post-stressing stage can be expressed as

$$\varepsilon_{pc,ps} = \varepsilon_{c,pl} - \varepsilon_{c,ps} \tag{13}$$

Hence, the strain of the steel plate in the post-stressing stage is

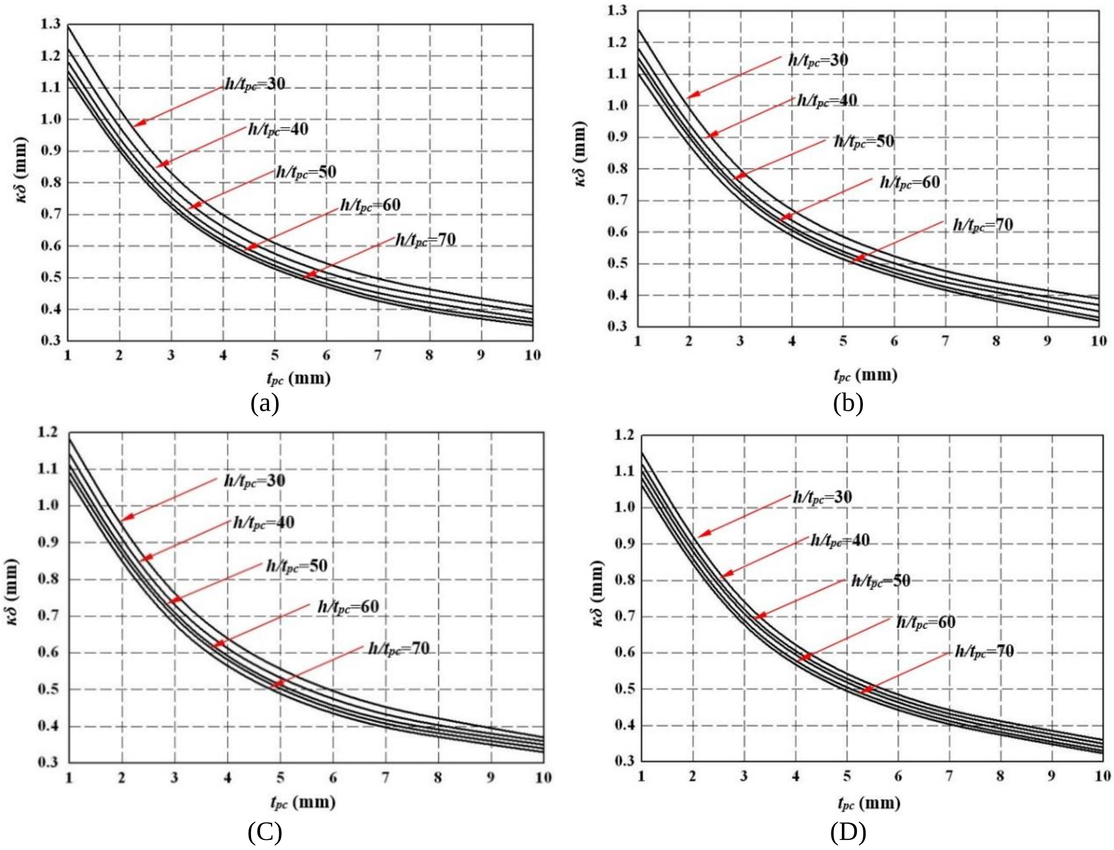
$$\varepsilon_{pc,ps} = \varepsilon_{c,pl} - \varepsilon_{cu} + \varepsilon_{pc} \tag{14}$$

Step 2. Checking the strain of the steel plate ($\varepsilon_{pc,ps}$) in the post-stressing stage

We assume that the changes in strain of the concrete is equal to that of the steel platesfrom post-stressing stage to ultimate stage. To maximize the load resistance of the strengthened column, the steel plates should yield before the occurrence of the compression failure of concrete. Hence, the strain of the steel plate ($\varepsilon_{pc,ps}$) in the post-stressing stage should satisfy Eq. (15), otherwise, the thickness of steel plates has to be increased.

$$\varepsilon_{pc,ps} \geq \varepsilon_{pcy} - \varepsilon_{c0} + \varepsilon_{c,pl} \tag{15}$$

The de-compressive force is controlled by the initial pre-camber of the steel plate. With the increase of the initial precamber, the de-compressive force in the RC column is increased. To prevent the concrete in tension, the strain of the steel plate ($\varepsilon_{pc,ps}$) in the post-stressing stage in the working load condition should also satisfy Eq. (16).



Note: $\kappa = 1/\sqrt{\dfrac{2\gamma_G L_{rc}{}^2 P'_{pc,ps}}{\pi^2 E_{pc} b}}$

Fig. 4 The relationship between plate thickness and initial pre-camber: (a) Ep = 10Ec; (b) Ep = 8Ec; (c) Ep = 6Ec and (d) Ep = 5Ec

$$\varepsilon_{pc,ps} \leq \frac{P_{pl}'}{E_{pc}\ A_{pc}} \tag{16}$$

Combining Eqs. (15) and (16), one can give

$$\frac{\varepsilon_{pcy} - \varepsilon_{c0} + \varepsilon_{c,pl}}{2} \leq \varepsilon_{pc,ps} \ \leq \ \frac{P_{pl}'}{E_{pc}\ A_{p\,c}} \tag{17}$$

Step 3. The post-compressive force in each steel plate

After obtaining the value of $\varepsilon_{pc,ps}$ from Eq. (14), the post-compressive force in the steel plate in the working condition can be calculated using Eq. (18)

$$\gamma_G P'_{pc,ps} \leq \chi_c t_{pc} h E_{pc} \varepsilon_{pc,ps} \tag{18}$$

<u>*Step 4. Initial pre-camber*</u>

The relationship between the plate thickness (t_{pc}) and the initial pre-camber (δ) can be expressed as (Su and Wang 2012, Wang and Su 2012a, b)

$$\delta^2 \geq \frac{2\gamma_G L_{rc}^2 P'_{pc,ps}}{\pi^2 t_{pc} h E_{pc}} + \frac{4\gamma_G L_{rc}^2 P'_{pc,ps}}{\pi^2 abE_c} \tag{19}$$

where a and b are the depth and width of the RC column, respectively; h is the width of the steel plate; L_{rc} is the clear height of the RC column; and E_c is the value for Young's modulus of the concrete.

To directly obtain the t_{pc} and δ values, a design chart is developed, as shown in Fig. 4.

2.5 Steel plates installed on the compression and tension faces of the RC column

In this section, a comprehensive design procedure for installing the steel plates on the compression and tension faces of the RC column is presented. The relationship among the initial pre-camber, plate thickness and depth of the compression zone in the ultimate stage will be described. The depth of the compression zone (c_{pl}) and the strain of the concrete ($\varepsilon_{c,pl}$) in the preloading stage can be obtained from Eqs. (4) - (7). The value of $\varepsilon_{pc,ps}$ can be checked using Eq. (15).

2.5.1 Determination of the required load capacity of the strengthened column in the working condition

<u>*Step 1. The original ultimate load capacity of the un-strengthened column*</u>

The vertical force equilibrium equation of the RC column can be obtained from the sum of the internal forces.

$$\gamma_G P'_{ori} \leq \varphi_t\,(\alpha\beta b c_{ori} f_c' + A_{sc} f_{scy} - A_{st}\, f_{sty}) \tag{20}$$

where P_{ori}' is the required load capacity of the un-strengthened column in the working condition and c_{ori} is the depth of the compression zone.

The moment equilibrium equation, which was obtained by taking moments about the tension steel, is given by

$$\gamma_G P'_{ori} e' \leq \varphi_t\,[\,\alpha\beta b c_{ori} f_c'(d - \frac{\beta c_{ori}}{2}) + A_{sc} f_{scy}(d - d')] \tag{21}$$

The P_{ori}' is the required load capacity of the un-strengthened column in the working condition (P_{ori}') can be obtained from Eqs. (4), (5), (20) and (21).

<u>*Step 2. Determining the size of the steel plates*</u>

The required load capacity of the strengthened column in the working condition (P_{pre}') is given by

$$\gamma_G P'_{pre} \leq \gamma_G P'_{ori} + t_{pc} b_{pc}\, E_{pc} \varepsilon_{pc}\, \chi_c - t_{pt} b_{pt}\, E_{pt} \varepsilon_{pt}\, \chi_c \tag{22}$$

where t_{pt} and b_{pt} are, respectively, the thickness and width of the tension steel plate, and ε_{pt} is the strain of the tension steel plate in the ultimate load stage. When the tension and compression steel plates yield at the same time, the size of the steel plates can be determined by replacing $\varepsilon_{pc}$and $\varepsilon_{pt}$with ε_{pcy} and ε_{pty}in Eq. (22), respectively.

2.6 Determination of the initial pre-camber of the steel plates

Step 1. Calculation of the height of the RC column ($L_{rc,pl}$)at the compression side during preloading

After obtaining the depth of the compression zone (c_{pl}) and the strain of the concrete ($\varepsilon_{c,pl}$) in the preloading stage from Eqs. (4) - (7), the height of the RC column ($L_{rc,pl}$)at the compression side during preloading can be obtained according to Fig. 5(a),

$$L_{rc,pl} = 2\left(\frac{c_{pl}}{\varepsilon_{c,pl}} - c_{pl}\right)\sin\left(\frac{L_{na}\varepsilon_{c,pl}}{2c_{pl}}\right) \tag{23}$$

where L_{na} is the height of the RC column at the neutral axis position, $c_{pl}/\varepsilon_{c,pl}$ is the radius of curvature of the column.

Step 2. The post-compressive force in the steel platein the working condition

The required load capacity of the PCP-strengthened RC column in the working condition (P_{pre}') is resisted by both the RC column and the steel plates. The vertical force equilibrium equation of the RC column can be obtained from the sum of the internal forces.

$$\gamma_G P_{pre}' \le \varphi_t\,(\alpha\beta b c_u f_c' + A_{sc} f_{scy} - A_{st}\ f_{sty}) + A_{pc} E_{pc}\left(\varepsilon_{pcy} - \varepsilon_{pc,ps}\right)\chi_c - A_{pt} f_{pt}\,\chi_t \tag{24}$$

The moment equilibrium Eq., which was obtained by taking moments about the tension steel, is given by

$$\gamma_G P_{pre}'\, e' \le \varphi_t\,[\alpha\beta b c_u f_c'\,(d - \tfrac{\beta c_u}{2}) + A_{sc}\, f_{scy}(d - d')] + \chi_c\, A_{pc}\, E_{pc}\left(\varepsilon_{pcy} - \varepsilon_{pc,ps}\right)(d + \tfrac{t_{pc}}{2}) - \chi_t\, A_{pt}\, f_{pt}\,(h - d + \tfrac{t_{pc}}{2}) \tag{25}$$

After obtaining the value of the strain of a pre-cambered steel plate ($\varepsilon_{pc,ps}$) at the post-stressing stage, the post-compressive force ($P_{pc,ps}'$) in the compression steel plate in the working condition can be calculated using Eq. (18). Meanwhile, $\varepsilon_{pc,ps}$ should satisfy Eq. (15),otherwise, thicker steel plates have to be used.

Step 3. Calculation of the depth of the compression zone (c_{ps}) and the concrete compressive strain ($\varepsilon_{c,ps}$) in the post-stressing stage

By considering the equivalent rectangular stress block, the equilibrium equation of the strengthened column can be obtained from the sum of the internal forces.

$$\gamma_G P_{pl}' \le \varphi_t\,[\alpha\beta b c_{ps} f_c'\left(\frac{2\varepsilon_{c,ps}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,ps}^{2}}{\varepsilon_{c0}}\right) + E_{sc}\ A_{sc}\varepsilon_{c,ps}\left(\frac{c_{ps} - d'}{c_{ps}}\right) - E_{st}\ A_{st}\ \varepsilon_{c,ps}\left(\frac{d - c_{ps}}{c_{ps}}\right)] + \gamma_G P_{pc,ps}' \tag{26}$$

The equation obtained from taking moments about the tension steel is

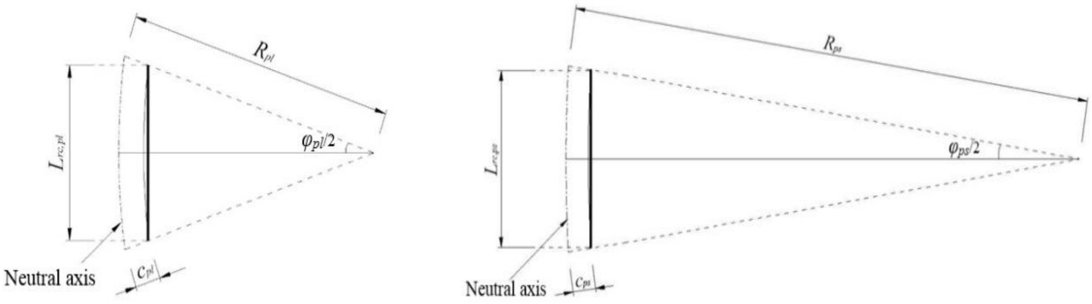


Fig. 5 Column curvature profile: (a) Preloading stage and (b) Post-stressing stage

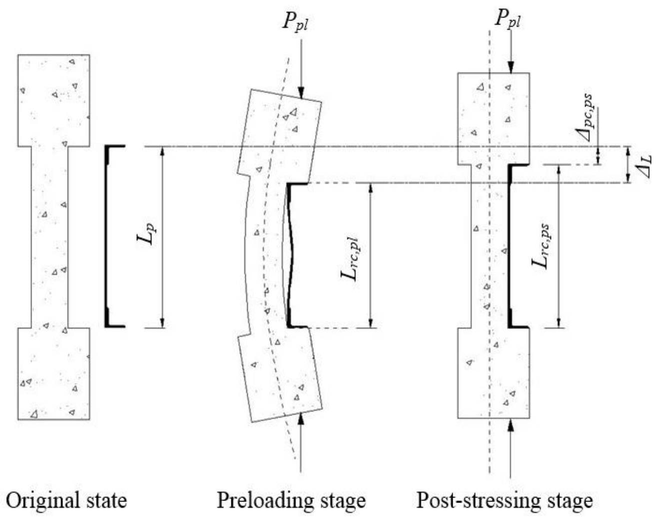


Fig. 6 Deformations of RC column under equivalent axial compression loading and bending moment

$$\gamma_G P_{pl}^{'} e' \leq \varphi_t [\alpha\beta b c_{ps} f_c{}' \left(\frac{2\varepsilon_{c,ps}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,ps}{}^2}{\varepsilon_{c0}}\right)(d - \frac{\beta c_{ps}}{2}) + E_{sc} A_{sc} \varepsilon_{c,ps} (\frac{c_{ps} - d'}{c_{ps}})(d - d') + \gamma_G P_{pc,ps}^{'} (\frac{t_{pc}}{2} + d) \tag{27}$$

The depth of the compression zone (c_{ps}) and strain of the concrete ($\varepsilon_{c,ps}$) in the post-stressing stage can be obtained by solving Eqs. (26) and (27).

According to Fig. 5(b), the height of the RC column ($L_{rc,ps}$) at the compression side after the flattening of the plate can be obtained using Eq. (28).

$$L_{rc,ps} = 2\,(\frac{c_{ps}}{\varepsilon_{c,ps}} - c_{ps})\sin(\frac{L_{na}\varepsilon_{c,ps}}{2c_{ps}}) \tag{28}$$

where L_{na} is the height of the RC column at the neutral axis position.

By considering the displacement compatibility of the steel plate and the RC column on the compression side, the plate axial shortening ($\Delta_{pc,ps}$) in the post-stressing stage can be estimated by Eq. (29) (see Fig. 6).

$$\Delta_{pc,ps} = L_{rc,pl} + \Delta_L - L_{rc,ps} \tag{29}$$

where Δ_L is the difference in the length of the steel plate and the RC column. According to the approximate precambered profile described withcosine function (Su and Wang 2012), Δ_L is found to be

$$\Delta_L = \frac{(\pi\delta)^2}{4L_{rc,pl}} \tag{30}$$

Step 4. Thickness of the compression steel plate and theinitial pre-camber

After determining the deformed height of the RC column ($L_{rc,ps}$) at the compression side when the plate is flattened, substituting Eqs. (23), (28) and (29) into Eq. (30), the relationship between the thickness of the compression steel plate and initial pre-camber (δ) can be expressed as

$$\delta \geq \frac{2}{\pi}\sqrt{L_{rc,pl}L_{rc,ps} - L_{rc,pl}{}^2 + \frac{\gamma_G P'_{pc,ps} L_{rc,pl} L_{rc}}{E_{pc} h t_{pc}}} \tag{31}$$

where the post-compressive force ($P_{pc,ps}$') in the steel plate in the working condition can be calculated using Eq. (18). The plate strain ($\varepsilon_{pc,ps}$) can be obtained from Eqs. (24) and (25).

Step 5. The maximum initial pre-camber of the compression steel plate

The decompressive force ($P_{pc,ps}$) is controlled by the initial pre-camber and thickness of the steel plate. With the increase of the initial pre-camber, the decompressive force in the RC column increases. To avoid the occurrence of a reversed moment in the RC column, the magnitude of the decompressive force should satisfy Eq. (32).

$$P_{pc,ps}\left(\frac{t_{pc}}{2} + d\right) \leq P'_{pl} e' \tag{32}$$

Substituting Eqs. (23), (28), (29) and (30) into Eq. (32), the maximum initial pre-camber (δ_{max}) of the compression steel plate should satisfy Eq. (33).

$$\delta_{max} \leq \frac{2}{\pi}\sqrt{\frac{E_{pc}A_{pc}\left(\frac{t_{pc}}{2} + d\right)L_{rc,pl}L_{rc,ps}}{E_{pc}A_{pc}\left(\frac{t_{pc}}{2} + d\right) - \gamma_G P'_{pl} e'} - L_{rc,pl}{}^2} \tag{33}$$

3. Design of vertical spacing of bolts

To prevent localized buckling of the steel plates, thecritical stress point ($\sigma_{p,cr}$) of the steel plates should be greater than the yield stress of the steel plates. Hence, the maximum bolt spacing for preventing localized plate buckling can be determined by Eq. (34).

$$\sigma_{p,cr} = \frac{G\pi^2 D}{h^2 t_{pc}} \geq f_{pcy} \tag{34}$$

where G is a bucking coefficient (Timoshenko and Gere 1961), the magnitude of which depends on the ratio of s_{max}/h and the edge restraining conditions, and D is the bending stiffness of the steel plate, which can be determined using Eq. (35).

$$D = \frac{E_{pc} t_{pc}^{\ 3}}{12(1-v^2)} \tag{35}$$

where v is Poisson's ratio of the steel plate.
Hence, the minimum value of G is

$$G = \frac{h^2\ t_{pc} f_{pcy}}{\pi^2 D} \tag{36}$$

The maximum bolt spacing (s_{max}) can be obtained according to the minimum value of G (Timoshenko and Gere 1961).

4. Design of the end bearing of the steel plates

4.1 Design ultimate bearing stress

According to ACI 318-02 (2001), for a bedding bearing on concrete, as shown in Fig. 7, the designed ultimate bearing stress should satisfy Eq. (37).

$$\gamma_G P'_{pc,ps} / h_b w \leq \varphi_b\ (0.85 f_c') \tag{37}$$

where h_b is the depth of the bedding bearing, w is the width of the bedding bearing, and φ_b is strength reduction factor. For the bearing on concrete, φ_b is equal to 0.65. From Eq. (37), the minimum width of the compression zone (w) can be obtained

4.2 Estimation of the shear capacity of the RC beam

For the existing RC beams, the shear capacity of the RC beam can be determined according to ACI 318-02 (2001).

The shear strength resisted by concrete is

$$V_c = 0.17\sqrt{f_c'}\, b_w d_e \tag{38}$$

where b_w is the web width of the cross section, and d_e is the effective depth, which is assumed to be equal to the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement.

The shear strength resisted by the shear reinforcement is

$$V_s = A_v f_{yh} (\, d_e / s_t\,) \tag{39}$$

where A_v, f_{yh}, and s_t are the total cross sectional area, the yield strength and the spacing of the transverse reinforcement, respectively.

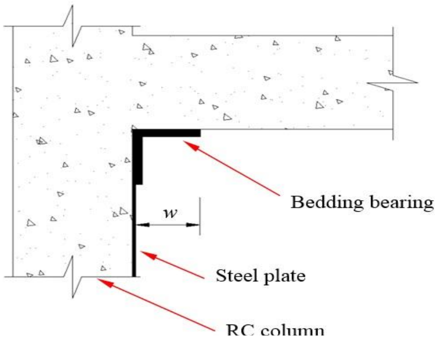


Fig. 7 The end detail of the steel plate

The shear capacity of the RC beam is

$$V_{rc} = V_c + V_s \tag{40}$$

To prevent the concrete from crushing at the joint, the post-compressive force in the compression steel plate should satisfy Eq. (41).

$$\gamma_G(P_{pl}' - P_{pc,ps}') < \varphi_s V_{rc} \tag{41}$$

5. Calculation of the tightening force of the bolts

The tightening force of the bolts (P_{tf}) can be calculated based on the principle of minimum total potential energy. The internal strain energy of the steel plate (U_{se}) is

$$U_{se} = \frac{E_{pc} A_{pc}}{2} \varepsilon_{pc,ps}^2 L_{pc} \tag{42}$$

where L_{pc} is the length of an un-deformed steel plate, and $\varepsilon_{pc,ps}$is a function of δ.

The potential energy of the tightening force (U_{tf}) is

$$U_{tf} = P_{tf}\delta \tag{43}$$

Then, the total potential energy is

$$E = U_{se} - U_{tf} \tag{44}$$

The equilibrium equation is given by

$$\frac{dE}{d\delta} = 0 \tag{45}$$

Hence, the tightening force (P_{tf}) can be obtained by solving Eqs. (42) - (45),

$$P_{tf} = \frac{E_{pc} A_{pc} L_{pc}}{2} \frac{d[\varepsilon_{pc,ps}^2(\delta)]}{d\delta} \tag{46}$$

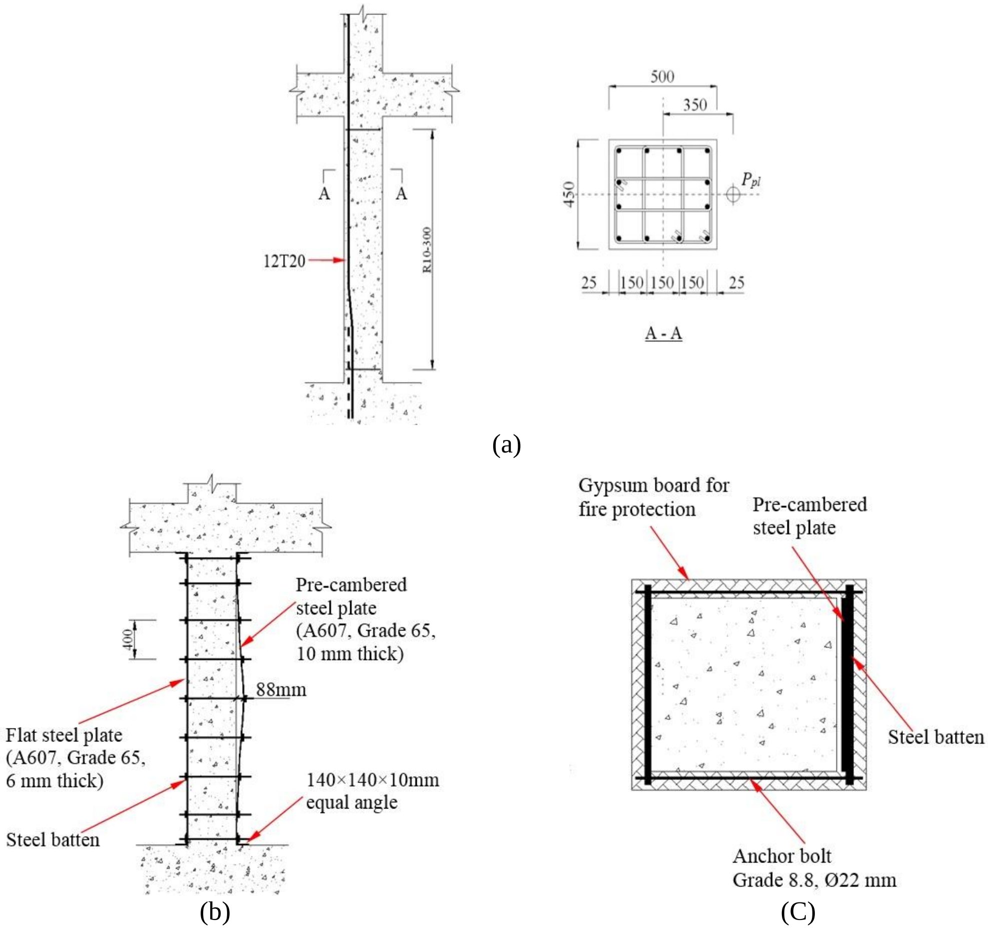


Fig. 8 Worked example (a) column details, (b) strengthening details and (c) batten details

For Case 1, substitutingEq. (19) into Eq. (46), the tightening force for the i^{th} buckling mode can be written as

$$P_{tf,i} = \frac{i\delta^3 E_{pc} A_{pc} L_{pc}}{2^i \left(\frac{L_{rc}}{\pi}\right)^4 \left(1 + \frac{2E_{pc}A_{pc}}{E_C A_C}\right)^2} \tag{47}$$

For Case 2, substituting Eq. (31) into Eq. (46), the tightening force for the i^{th} buckling mode can be written as

$$P_{tf,i} = \frac{iE_{pc} A_{pc} L_{pc}\left[\pi^4 \delta^3 + \pi^2 \delta \left(L_{rc,pl}{}^2 - L_{rc,pl} L_{rc,ps}\right)\right]}{2^{i+2} \left(\frac{L_{rc}}{\pi}\right)^4 \left(L_{rc,pl} L_{rc}\right)^2} \tag{48}$$

After obtaining the tightening force, the design value ($P_{dtf,i}$) can be expressed as

$$P_{dtf,i} \geq \gamma_G P_{tf,i} \tag{49}$$

6. Worked example

In this example, a 450 mm (a) ×500 mm (b) rectangular RC column is subjected to 800 kN of eccentric compression loading (P_{pl}) in the working condition, with an eccentricity (e) of 350 mm. The RC column is to be strengthened with pre-cambered and flat steel plates. The required load capacities of the strengthened column in the working and ultimate load conditions are1800 kN and 2520 kN respectively. The load factor for gravity load is 1.4 .The column clear height (L_{rc}) is 3000 mm. A vertical reinforcement of 12T20 was arranged, and a transverse reinforcement of R10 - 300 was applied throughout the height of the column. The yield stress (f_{sy}) and Young's modulus (E_s) of the longitudinal reinforcement are 500MPa and 200GPa, respectively. The yield stress (f_{yh}) and Young's modulus (E_{ts}) of the transverse reinforcement are 350 MPa and 195 GPa, respectively. The cross section of the beam is 450 mm × 500 mm (depth). The reinforcement details are shown in Fig. 8(a). The cube and cylinder strength of the concrete are 50 MPa (f_{cu}) and 40 MPa (f_c'), respectively. The strength reduction factor of bearing on the concrete (φ_b) is 0.65; the strength reduction factor of the tension-controlled sections (φ_t) is 0.9; the strength reduction factor of the compression-controlled sections (φ_c) is 0.65. For the steel plate, both safety factors for compression (χ_c) and tension (χ_t) are 0.9; and the partial safety factor for dead loads is (γ_G) is 1.4.

6.1 Estimating the eccentricity of the RC column

Before choosing the type of strengthening, the eccentricity of the RC column corresponding to the balance failure should be calculated. In this example, ε_{co} and ε_{cu} are equal to 0.002 and 0.003, respectively (Park and Paulay 1975). The depth of the compression zone (c_b) of the concrete at balanced failure can be obtained using Eq. (1)

$$c_b = \frac{(500-25)\times 0.003\times 200000}{500\times 0.003\times 200000} = 259 \text{ mm}$$

According to Eqs. (4) and (5), the values of α and β can be obtained.

$$\alpha = [\frac{0.003}{0.002} - \frac{1}{3}(\frac{0.003}{0.002})^2]/0.833 = 0.9$$

$$\beta = (4 - \frac{0.003}{0.002})/(6 - \frac{2\times 0.003}{0.002}) = 0.833$$

Substituting the value of c_b into Eqs. (2) and (3), the compression loading (P_b) and eccentricity (e_b) at balanced failure can be calculated.

$$P_b = 0.833\times 0.9\times 259\times 450\times 40 = 3495 \text{ mm}$$

$$3495\times 10^3\times(e_b + \frac{500}{2} - 25) = 0.833\times 0.9\times 259\times 450\times 40\times$$

$$(475 - \frac{0.833\times 259}{2}) + 12\times 3.14\times 10^2\times 350\times(475-25)$$

Solving the above equation

$$e_b = 311 \text{ mm}$$

Because $e > e_b$, the flat and pre-cambered steel plates, which were placed on the tension and compression sides of the RC column, respectively, were used to strengthen the RC column (Case 2).

6.2 The depth of the compression zone (c_{pl}) *and the* strain of the concrete ($\varepsilon_{c,pl}$) in the *preloading stage*

The equilibrium equation of the RC column can be obtained from the sum of the internal forces

$$P_{pl}' = (0.833 \times 0.9 \times 450 \times c_{pl} - 314 \times 6) \times 40 \times \left(\frac{2\varepsilon_{c,pl}}{0.002} - \frac{\varepsilon_{c,pl}^{2}}{0.002^{2}}\right) + 200000 \times 3.14 \times 10^{2} \times 4 \times \varepsilon_{c,pl}$$

$$\times \left(\frac{c_{pl} - 25}{c_{pl}}\right) + 200000 \times 3.14 \times 10^{2} \times 2 \times \varepsilon_{c,pl} \times \left(\frac{c_{pl} - 150\text{-}25}{c_{pl}}\right) - 200000 \times 3.14 \times 10^{2} \times 4 \times \varepsilon_{c,pl}$$

$$\times \left(\frac{475 - c_{pl}}{c_{pl}}\right) - 200000 \times 3.14 \times 10^{2} \times 2 \times \varepsilon_{c,pl} \times \left(\frac{475 - 150 - c_{pl}}{c_{pl}}\right) = 8\,00 \text{ kN}$$

The equation obtained from taking moments about the side tension steel is

$$P_{pl}'e' = (0.833 \times 0.9 \times 450 \times c_{pl} - 314 \times 6) \times 40 \times \left(\frac{2\varepsilon_{c,pl}}{0.002} - \frac{\varepsilon_{c,pl}^{2}}{0.002^{2}}\right)\left(475 - \frac{0.833 \times c_{pl}}{2}\right) + 200000$$

$$\times 3.14 \times 10^{2} \times 4 \times \varepsilon_{c,pl} \times \left(\frac{c_{pl} - 25}{c_{pl}}\right) \times (475 - 25)$$

$$+ 200000 \times 3.14 \times 10^{2} \times 2 \times \varepsilon_{c,pl} \times (475 - 150 - 25) - 200000 \times 3.14 \times 10^{2} \times 4 \times \varepsilon_{c,pl} \times 150$$

$$= 800 \times 575 \text{ kN mm}$$

Solving the above equations

$$\varepsilon_{c,pl} = 0.0006$$

$$c_{pl} = 163 \text{ mm}$$

Because $e > e_b$, the flat and pre-cambered steel plates, which were placed on the tension and compression sides of the RC column, respectively, were used to strengthen the RC column (Case 2).

6.2 The required load capacity of the original column in the working condition

The equilibrium equation of the RC column can be obtained from the sum of the internal forces

$$1.4 \times P_{ori}' \leq 0.9 \times (0.833 \times 0.9 \times 45 \times 450 \times c_{ori} + 3.14 \times 6 \times 40 \times 10^{2} \times 500 - 3.14 \times 6 \times 40 \times 10^{2} \times 500)$$

The equation obtained from taking moments about the side tension steel is

$$1.4 \times P_{ori}' \times (350 + 225) \leq 0.9 \times [0.833 \times 0.9 \times 45 \times 450 \times c_{\text{ori}} \times 40 \times \left(475 - \frac{0.833 \times c_{ori}}{2}\right) + 3.14 \times 4 \times 10^{2}$$

$$\times 500 \times (475 - 25) + 3.14 \times 2 \times 10^{2} \times 500 \times (475 - 150 - 25) - 3.14 \times 2 \times 10^{2} \times 500 \times (475 - 150 \times 2 - 25)]$$

Solving the above equations, the ultimate load capacity of the original column can be obtained.

$$P'_{ori} = 1300\ \text{kN},\ c_{ori} = 150\ \text{mm}$$

6.3 Determining the size of steel plates

According to Eq. (22), the axial load resistance provided by steel plates is

$$\chi_c t_{pc} b_{pc} E_{pc} \varepsilon_{pcy} - \chi_t t_{pt} b_{pt} E_{pt} \varepsilon_{pty} \geq 1.4 \times \left(P'_{pre} - P'_{ori}\right) = 1.4 \times (1800 - 1300) = 700\ \text{kN}$$

Try A607 Grade 65 steel, the yield stress (f_{pcy}) and Young's modulus (E_{pc}) of the steel plates are 450 MPa and 198 GPa, respectively. The width of steel plates is 450 mm and $\chi_c = \chi_t = 0.9$

Try t_{pc} = 10 mm and t_{pt} = 6 mm of A607 Grade 65 steel plates

The axial capacity of the steel plates is $405 \times \left(t_{pc} E_{pc} \varepsilon_{pcy} - t_{pt} E_{pt} \varepsilon_{pty}\right) = 729\ \text{kN} > 700\ \text{kN}$ Okay

6.4 The height of the RC column (Lrc,pl)at the compression side during preloading

The height of the RC column ($L_{rc,pl}$)at the compression side during preloading can be calculated using Eq. (23)

$$L_{rc,pl} = 2\left(\frac{c_{pl}}{\varepsilon_{c,pl}} - c_{pl}\right) \sin\left(\frac{L_{na} \varepsilon_{c,pl}}{2 c_{pl}}\right)$$

$$= 2\left(\frac{163}{0.0006} - 163\right) \sin\left(\frac{3000 \times 0.0006}{2 \times 163}\right)$$

$$= 2996\ \text{mm}$$

6.5 The required load capacity of the strengthened column in the working condition

The equilibrium equation of the RC column can be obtained from the sum of the internal forces

$$1.4 \times P_{pre}' \leq 0.9 \times [(0.833 \times 0.9 \times 450 \times c_u - 3.14 \times 6 \times 10^2) \times 40 + 500 \times 6 \times 3.14 \times 10^2 - 500 \times 6 \times 3.14 \times 10^2)]$$

$$+ 0.9 \times 10 \times 450 \times 198000 \times (0.0022 - \varepsilon_{pc,ps}) - 0.9 \times 8 \times 450 \times 198000 \times (0.0022 - \varepsilon_{pc,ps}) \times \left(\frac{500 - c_u + 3}{c_u + 5}\right)$$

$$= 1.4 \times 1800\ \text{kN}$$

The equation obtained from taking moments about the side tension steel is

$$1.4 \times P_{pre}'e' \leq 0.9 \times [(0.833 \times 0.9 \times 450 \times c_u - 3.14 \times 6 \times 10^2) \times 40 \times \left(475 - \frac{0.833 \times c_u}{2}\right) + 500 \times 4 \times 3.14 \times 10^2$$

$$\times (475-25) + 500 \times 2 \times 3.14 \times 10^2 \times (475-150-25) - 500 \times 2 \times 3.14 \times 10^2 \times 150)] + 0.9 \times 10 \times 450 \times 198000$$

$$\times (0.0022 - \varepsilon_{pc,ps}) \times (475 + 5) - 0.9 \times 8 \times 450 \times 198000 \times (0.0022 - \varepsilon_{pc,ps})$$

$$\times \left(\frac{500 - c_u + 3}{c_u + 5}\right) \times (25 + 3) = 1.4 \times 1800 \times 575\ \text{kNmm}$$

The strain of the compression steel plate at the post-stressing stage is

$$\varepsilon_{pc,ps} = 0.0016,\ c_u = 207\ \text{mm}$$

Checking the value of $\varepsilon_{pc,ps}$ using Eq. (15)

$$\varepsilon_{pc,ps} \geq \varepsilon_{pcy} - \varepsilon_{co} + \varepsilon_{c,pl} = 0.0008 \text{ \underline{Okay}}$$

6.6 The post-compressive force in the compression plate in the working condition

$$P_{pc,ps}' = \chi_c t_{pc} h E_{pc} \varepsilon_{pc,ps}/\gamma_G = 0.9 \times 450 \times 10 \times 198000 \times 0.0016/1.4 = 916 \text{ kN}$$

6.7 The depth of the compression zone ($c_{c,ps}$) and the concrete compressive strain ($\varepsilon_{c,ps}$) in the post-stressing stage

The equilibrium equation of the RC column can be obtained from the sum of the internal forces

$$\begin{aligned}
1.4 \times P_{pl}' \leq\ & 0.9 \times [(0.833 \times 0.9 \times 450 \times c_{ps} - 314 \times 6) \times 40 \\
& \times \left(\frac{2\varepsilon_{c,ps}}{0.002} - \frac{\varepsilon_{c,ps}^2}{0.002^2}\right) + 200000 \times 3.14 \times 10^2 \times 4 \times \varepsilon_{c,ps} \\
& \times \left(\frac{c_{ps} - 25}{c_{ps}}\right) + 200000 \times 3.14 \times 10^2 \times 2 \times \varepsilon_{c,ps} \times \left(\frac{c_{ps} - 150 - 25}{c_{ps}}\right) \\
& - 200000 \times 3.14 \times 10^2 \times 4 \times \varepsilon_{c,ps} \times \left(\frac{475 - c_{ps}}{c_{ps}}\right) \\
& - 200000 \times 3.14 \times 10^2 \times 2 \times \varepsilon_{c,ps} \times \left(\frac{475\text{-}150\text{-}c_{ps}}{c_{ps}}\right)] + 1.4 \times 916000 \\
& = 1.4 \times 800 \text{ kN}
\end{aligned}$$

The equation obtained from taking moments about the side tension steel is

$$\begin{aligned}
1.4 \times P_{pl}' \times 575 \leq\ & 0.9 \times [(0.833 \times 0.9 \times 450 \times c_{ps} - 314 \times 6) \times 40 \times \left(\frac{2\varepsilon_{c,ps}}{0.002} - \frac{\varepsilon_{c,ps}^2}{0.002^2}\right) \times \left(475 - \frac{0.833 \times c_{ps}}{2}\right) \\
& + 200000 \times 3.14 \times 10^2 \times 4 \times \varepsilon_{c,ps} \times \left(\frac{c_{ps} - 25}{c_{ps}}\right) \times (475\text{-}25) + 200000 \times 3.14 \times 10^2 \times 2 \times \varepsilon_{c,ps} \times \left(\frac{c_{ps} - 150\text{-}25}{c_{ps}}\right) \\
& \times (475\text{-}150\text{-}25) - 200000 \times 3.14 \times 10^2 \times 4 \times \varepsilon_{c,ps} \times 150] + 1.4 \times 916000 \times (475+5) \\
& = 1.4 \times 800 \times 575 \text{ kNmm}
\end{aligned}$$

Solving the above equations

$$\varepsilon_{c,ps} = 0.0002$$

$$c_{ps} = 84 \text{ mm}$$

The deformed height of the RC column ($L_{rc,ps}$) at the compression side after flattening the pre-cambered steel plate can be obtained using Eq. (28),

$$L_{rc,ps}=2(\frac{c_{ps}}{\varepsilon_{c,ps}}\text{-}c_{ps})\sin(\frac{L_{na}\varepsilon_{c,ps}}{2c_{ps}})$$
$$=2(\frac{84}{0.0002}\text{-}84)\sin(\frac{3000\times0.0002}{2\times84})$$
$$=2998\text{ mm}$$

6.8 Thickness of the compression steel plate and initial pre-camber

Eq. (31), the initial pre-camber displacement is

$$\delta\geq\frac{2}{\pi}\sqrt{L_{rc,pl}L_{rc,ps}-L_{rc,pl}{}^{2}+\frac{\gamma_G P'_{pc,ps}L_{rc,pl}L_{rc}}{E_{pc}ht_{pc}}}=$$
$$\frac{2}{\pi}\sqrt{2998\times2996\text{-}2996^{2}+\frac{1.4\times916000\times2998\times3000}{198000\times450\times10}}=88\text{ mm}$$

6.9 Checking the maximum initial pre-camber of the compression steel plate

(33), the maximum initial pre-camber displacement is

$$\delta\ \leq\frac{2}{\pi}\sqrt{\frac{E_{pc}A_{pc}(\frac{t_{pc}}{2}+d)L_{rc,pl}L_{rc,ps}}{E_{pc}A_{pc}(\frac{t_{pc}}{2}+d)\text{-}\gamma_G P'_{pl}e'}-L_{rc,pl}{}^{2}}$$
$$=\frac{2}{\pi}\sqrt{\frac{198000\times450\times10\times(\frac{10}{2}+475)\times2996\times2998}{198000\times450\times10\times(\frac{10}{2}+475)\text{-}1.4\times800\times10^{3}\times575}-2996^{2}}=90\text{ mm}$$

<u>Use initial camber of 88 mm at the mid-height of the steel plate</u>

6.10 The vertical spacing of the bolts(s)

According to Eqs. (34) - (36), the critical stress point of the steel plate can be calculated

$$D=\frac{E_{pc}t_{pc}{}^{3}}{12(1-v^{2})}=\frac{198000\times10^{3}}{12\times(1\text{-}0.25^{2})}=176\times10^{5}$$
$$\sigma_{p,cr}=\frac{G\pi^{2}D}{h^{2}t_{pc}}=\frac{\text{G}\times3.14^{2}\times176\times10^{5}}{450^{2}\times10}\geq450$$

Hence, $G\geq5.25$.
The ratio of s_{max}/h is 0.9 if the minimum value of G is equal to 5.25.
Then, the maximum vertical spacing of the bolts is
$S_{max}=0.9\times450=405$ mm <u>Use thebolt spacing of 400 mm</u>

6.11 Design ultimate bearing stress

According to Eq. (37), for the components under shear force, φ_b is equal to 0.65, and the minimum width of the compression zone (w) is

$$w > \frac{\gamma_G P'_{pc,ps}}{h_b \varphi_b (0.85 f_c')} = \frac{1.4 \times 916000}{450 \times 0.65 \times 40 \times 0.85} = 129 \text{ mm}$$

Use 140×140×10 equal angle

6.12 Maximum design shear stress

According to Eq. (38), the shear strength resisted by the concrete (V_c) is

$$V_c = 0.17\sqrt{f_c'} b_w d_e = 0.17 \times \sqrt{40} \times 300 \times 480 = 155 \text{ kN}$$

According to Eq. (39), the shear strength resisted by the shear reinforcement (V_s) is

$$V_s = A_v f_{yh} (d_e / s_t) = 314 \times 500 \times (480/100) = 528 \text{ kN}$$

Then, the shear capacity of the RC beam (V_{rc}) is

$$V_{rc} = V_c + V_s = 155 + 528 = 683 \text{ kN}$$

Because the balance force in the steel plate is less than the shear capacity of the RC beam

$$\sum P = 1.4 \times \left(P'_{pc,ps} - P'_{pl}\right) = 1.4 \times \left(916000 - 800000\right) = 162 \text{ kN} < 683 \text{ kN} \underline{\text{Okay}}$$

The initial pre-camber displacement and the size of the steel plates are feasible in this example.

6.13 Calculation of tightening force

The tightening force (P_{tf}) can be calculated based on the energy equilibrium. According to Eq. (49), the tightening force ($P_{tf,3}$) of the pre-cambered steel plate for the third buckling mode (i.e., i=3) is

$$P_{tf,3} = \frac{i E_{pc} A_{pc} L_{pc} \left[\pi^4 \delta^3 + \pi^2 \delta (L_{rc,pl}{}^2 - L_{rc,pl} L_{rc,ps})\right]}{2^{i+2} \left(\frac{L_{rc}}{\pi}\right)^4 \left(L_{rc,pl} L_{rc}\right)^2}$$

$$= \frac{3 \times 198000 \times 4500 \times 3000 \times \left[3.14^4 \times 85^3 + 3.14^2 \times 85 \times (2996^2 - 2996 \times 2998)\right]}{2^{3+2} \times (2996 \times 3000)^2} = 161 \text{ Kn}$$

The design value of tightening force ($P_{dtf,3}$) is

$$P_{dtf,3} \geq \gamma_G P_{tf,3} = 1.4 \times 161 = 226 \text{ kN}$$

Use 2 nos. Grade 8.8 M22 bolts
(Tensile capacity = 252kN)

The strengthening details are shown in Fig. 8(b). The batten and anchor bolt details are shown in Fig. 8(c).

7. Conclusions

With the aim of improving the strength, deformation and ductility of existing RC columns, a new strengthening approach using post-compressed plates (PCP) was developed. The effectiveness and efficiency of this approach were demonstrated by extensive experimental and theoretical studies. In this paper, a comprehensive design procedure is proposed to design the PCP-strengthened RC columns. The proposed design procedure consists of five main parts, which are (1) determining the size of steel plates according to the requiredultimate load capacity of the strengthened column, (2) the design of the initial pre-camber displacement of the steel plate, (3) the design of the vertical spacing of the bolts, (4) the design of the bearing ends of the steel plates, and (5) the calculation of the tightening force of the bolts. With this design procedure, the initial pre-camber displacement of the steel plate and the minimum width of the compression zone at the plate ends can be determined to achieve the required ultimate load capacity of the columns. A worked example on the design of an RC column strengthened with post-compressed plates is offered to demonstrate the application of the proposed design procedure.

Acknowledgements

The research described here was supported by the Research Grants Council of the Hong Kong SAR (Project No. HKU7166/08E) and The University of Hong Kong through their Small Project Funding 2010-2011.

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