Photo-thermo-elastic interaction in a semiconductor material with two relaxation times by a focused laser beam

A. Jahangir¹, F. Tanvir¹ and A. M. Zenkour²,³

¹Department of Mathematics, COMSATS University Islamabad, Wah Campus 47040, Pakistan
²Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
³Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafrelsheikh 33516, Egypt

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Abstract. The effect of relaxation times is studied on plane waves propagating through semiconductor half-space medium by using the eigen value approach. The bounding surface of the half-space is subjected to a heat flux with an exponentially decaying pulse and taken to be traction free. Solution of the field variables are obtained in the form of series for a general semiconductor medium. For numerical values, Silicon is considered as a semiconducting material. The results are represented graphically to assess the influences of the thermal relaxations times on the plasma, thermal, and elastic waves.

Keywords: semiconductor material; eigen value approach; elastic waves; focused laser beam; generalized thermoelastic theories

1. Introduction

Semiconductors with a band gap energy are considered by means of a laser beam with an energy $E$ greater than that of $E_g$, then an excitation process occurs. The electrons from the valence range are transferred to the energy level and their energy $(E - E_g)$ is greater than the edge of the conductivity band. These free carriers will relax to an unfilled level near the lower of the conduction band. After the relaxation process, the electron and hole plasma are found, followed by pairs of electron holes formed by the recombination process. Local deformation can cause local tensions in the sample, which provides a plasma wave like the thermal wave resulting from local periodic elastic deformation. The semiconductors materials are useful in radar and microwave similarly semiconductors are also used in very-high-speed SiGe devices.

In the last 40 years, the thermal theory has been very interested in the finite speed of thermal signals. These theories are called general thermoelastic theory. Lord and Shulman (1967) both proposed the first general theory of thermoelasticity which involved one relaxation time, while Green and Lindsay (1972) had a second general theory of thermal heat with two relaxation times. On the contrary, the coupled thermoelastic theory is associated with the parabolic heat equation.

Experimental and theoretical analysis of plasma, thermal and micromechanical in one dimension was made by Todorović et al. (2003a, b) to deal with the properties of carrier
recombination and transport in the semiconducting material. In addition, such study includes the propagation of changes in heat and plasma waves due to the linear coupling between heat and mass transfer. Opsal and Rosencwaig (1985) and Rosencwaig et al. (1983) studied the depth of thermal and plasma waves in silicon. As an important branch of the mechanical properties of solids, the literature addresses various problems through numerical and analytical methods. On the other side, Song et al. (2010) studied the thermoelastic vibration produced by the optically excited semiconducting microcantilevers. They concluded that, the wave reflection in a semiconducting plane under photo thermal and theories of generalized thermoelasticity. Kumar and Vohra (2017) studied the vibration analysis of thermoelastic double porous microbeam subjected to laser pulse. They used the Laplace and Fourier’s transformation to find the solution of the problem. Kumar et. al. (2016a, b), Sharma and Sharma (2014) are containing some important results related to wave propagation phenomena.

The photothermal waves in a one-dimensional semiconductor medium is studied by Abbas et al. (2017) and Hobiny and Abbas (2017). In Laplace, the eigen value method gives an analytical solution without any supposed restriction on the actual physical quantities. Lotfy (2017) presented photothermal waves for two-temperature model with a semiconductor medium due to a dual-phase-lag theory and hydrostatic initial stress. In other recent articles, Zenkour (2018a, b, 2019a, b) presented a multi dual-phase-lag theory to treat the thermomechanical response of microbeams, the micro-temperatures for plane wave propagation in thermoelastic medium, and the photothermal waves of a gravitated semiconducting half-space.

In this article, the heat waves propagating through the body are analyzed by using heat conduction equation with two relaxation times by Green and Lindsay (1972) (see also, Sharma et al. 2008). An analytical technique of eigen value approach is used to study their effects on the waves.

2. Mathematical formulation

The efforts are made to study the plasma, thermal and elastic waves generated by a focused laser beam in an elastic medium. For simplicity, the surface of the medium $\mathbb{S}$ is supposed to be half space and $y$-axis is pointing vertically into the medium $\mathbb{S} = \{(x, y, z): -\infty \leq x \leq \infty, y \geq 0, -\infty \leq z \leq \infty\}$, with $z$-axis taken along the symmetry such that effects and changing do not appear along this axis. System of governing equations for the two-dimensional semiconductor under the influence of laser beam with radius $r$ (Todorović 2005, Mandelis et al. 1997) is represented as

$$\begin{align*}
(\lambda + \mu)u_{ij,j} + \mu u_{i,ij} - \gamma_\nu N_{,i} - \gamma_T \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \theta_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \\
D_{,i} N_{,j} = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \theta \frac{\partial \theta}{\partial t} + Q, \\
K \theta_{,j} = -\frac{E_a}{\tau} N + \rho c_v \left(1 + \tau_\nu \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial t} + \gamma_T \left(1 + \tau_T \frac{\partial}{\partial t}\right) \frac{\partial u_{,j}}{\partial t} + \gamma_T \frac{\partial \theta}{\partial t} + \delta_k Q,
\end{align*}$$

and the stress-strain relations are presented as

$$\sigma_{ij} = \mu (u_{i,j} + u_{j,i}) + \left[\lambda u_{kk} - \nu \nu N - \gamma_\nu \left(1 + \tau_\nu \frac{\partial}{\partial t}\right) \theta \right] \delta_{ij}. $$
where \( \rho \) denotes the density of medium, \( \sigma_{ij} \) are the stress components, \( u_i \) are the displacement components, \( \tau_0, \tau_1 \) are the thermal relaxation times (for semiconductor \( 10^{-12} \leq \tau_0 \leq 10^{-10} \) s, \( 10^{-12} \leq \tau_1 \leq 10^{-10} \) s), \( \theta = T - T_0 \), \( T_0 \) is the reference temperature, \( N = n - n_0 \), \( n_0 \) is the equilibrium carrier concentration, \( K \) denotes the thermal conductivity, \( \delta_E = E - E_g \) , \( E \) represents the excitation energy, \( E_g \) denotes the energy gap of the semiconductor, \( \lambda, \mu \) represent Lame's constants, \( c_e \) denotes the specific heat at constant strain, \( D_e \) denotes the carrier diffusion coefficient, \( \gamma_n = (3\lambda + 2\mu)d_n, d_n \) represents the coefficient of electronic deformation, \( \gamma_t = (3\lambda + 2\mu)\alpha_t \), \( \alpha_t \) represents the linear thermal expansion coefficient, \( \tau \) denotes the photogenerated carrier lifetime, \( \theta = \frac{\alpha_0}{\alpha_0} \) denotes the coupling parameter of thermal activation, and \( t \) is the time.

The plate surface is illuminated by a laser pulse (Othman et al. 2015, Zenkour and Abouelregal 2015, Abouelregal and Zenkour 2019) given as

\[
Q(x,y,t) = \frac{I_0 r^*}{2\pi r^* t_0^2} \exp \left(-\frac{x^2}{r^2} - \frac{t}{t_0}\right) \exp(-\gamma^* y),
\]

where \( I_0 \) denotes the energy absorbed, \( t_0 \) represents the pulse rise time, \( r \) represents the beam radius and \( \gamma^* \) denotes the absorption depth of heating energy.

Let us consider the state of plane strain in the present 2D problem of a semiconductor half-space. The variable components are defined by \( u_i \equiv (u,v,0), u \equiv u(x,y,t), v \equiv v(x,y,t), \theta \equiv \theta(x,y,t) \) and \( N \equiv N(x,y,t) \). Therefore, Eqs. (1)-(4) can be written by

\[
(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial x \partial y} + \mu \frac{\partial^2 u}{\partial y^2} + \gamma_n \frac{\partial N}{\partial x} - \gamma_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2},
\]

\[
(\lambda + 2\mu) \frac{\partial^2 v}{\partial y^2} + (\lambda + \mu) \frac{\partial^2 v}{\partial x \partial y} + \mu \frac{\partial^2 v}{\partial x^2} - \gamma_n \frac{\partial N}{\partial y} - \gamma_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2},
\]

\[
D_e \left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \frac{\partial \theta}{\partial t} + Q,
\]

\[
K \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = \frac{E_g}{\tau} N + \rho c_e \left(1 + \frac{\tau}{\tau_0}\right) \frac{\partial \theta}{\partial t} + \gamma_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \delta q \theta.
\]

At \( y = 0 \), the boundary conditions (Hobiny and Abbas, 2018) are considered by

\[
\sigma_{yy} = \sigma_{xx} = 0, \quad D_e \frac{\partial N}{\partial y} - s_0 N = 0, \quad -K \frac{\partial \theta}{\partial y} = \frac{q_0 t^2 e^{-t_0 / \tau}}{16 \mu},
\]

where \( s_0 \) represents the speed of surface recombination, \( q_0 \) denotes a constant and \( t_p \) denotes the characteristic time of the pulse heat flux.

It is convenient to transform the above equations into dimensionless forms. To do this, the dimensionless quantities can be introduced as

\[
(t^*, \tau^*, \tau_0^*, \tau_1^*) = \eta c^2 (t, \tau, \tau_0, \tau_1), \quad (x^*, y^*, u^*, v^*) = \eta c (x, y, u, v),
\]

\[
(\sigma_{xx}^*, \sigma_{yy}^*, \sigma_{xy}^*) = \frac{1}{\mu} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}), \quad N^* = \frac{N}{n_0}, \quad \theta^* = \frac{\theta}{\tau_0}, \quad Q^* = \frac{Q}{n_0 \eta c^2 D_e},
\]

where \( \eta = \frac{\rho c}{K} \) and \( c = \sqrt{\frac{\lambda + 2\mu}{\rho}} \).

By neglecting the asterisk and rewriting Eqs. (5)-(9), we obtain,
\[
\frac{\partial^2 u}{\partial x^2} + a_1 \frac{\partial^2 v}{\partial x \partial y} + a_2 \frac{\partial^2 u}{\partial y^2} - \beta_n \frac{\partial N}{\partial x} - \beta_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{12}
\]
\[
\frac{\partial^2 v}{\partial y^2} + a_1 \frac{\partial^2 u}{\partial x \partial y} + a_2 \frac{\partial^2 v}{\partial x^2} - \beta_n \frac{\partial N}{\partial y} - \beta_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{13}
\]
\[
\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} = \frac{\partial \partial N}{\partial t} + \frac{\partial N}{\partial t} - \beta \frac{\partial \theta}{\partial t} + Q_0 \frac{t}{t_0} \exp\left(\frac{-x^2}{r^2} - \frac{t}{t_0}\right) \exp(-\gamma'y), \tag{14}
\]
\[
\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = -\frac{\epsilon_1}{\tau} N + \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial t} + \frac{\epsilon_2}{\tau} \left(1 + m \tau_0 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t}\right) \tag{15}
\]
\[
\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} = \frac{\partial \partial N}{\partial t} + \frac{\partial N}{\partial t} - \beta \frac{\partial \theta}{\partial t} + Q_0 \frac{t}{t_0} \exp\left(\frac{-x^2}{r^2} - \frac{t}{t_0}\right) \exp(-\gamma'y). \tag{16}
\]
where
\[
\alpha_i = \frac{\lambda + \mu}{\lambda + 2\mu}, \quad \alpha_2 = \frac{\mu}{\lambda + 2\mu}, \quad \alpha_3 = \frac{\lambda}{\lambda + 2\mu}, \quad \beta_n = \frac{n_0}{n_0 + \gamma \eta r}, \quad \beta_t = \frac{T_0}{\lambda + 2\mu}, \quad \omega = \frac{1}{\eta \beta}, \quad \beta = \frac{\beta_0}{n_0 \beta y}, \quad \epsilon_1 = \frac{y_1}{\rho c_T}, \quad \epsilon_2 = \frac{y_2}{\rho c_T}, \quad Q_0 = \frac{E n_0}{\eta K T_0}, \quad Q_1 = \frac{\delta \tau_0 D_N (l_0 y')}{K T_0}. \tag{17}
\]

3. Harmonic solution

The solution of the considered physical quantities can be decomposed in terms of normal mode analysis as
\[
[u, v, N, \theta, \sigma_{ij}] = [u^*, v^*, N^*, \theta^*, \sigma_{ij}^*] f(y) e^{i(\alpha x - \omega t)}, \tag{18}
\]
where \( u^*, v^*, N^*, \theta^* \) and \( \sigma_{ij}^* \) are the wave number in the \( x \)-direction, \( i = \sqrt{-1} \) and \( \omega \) is the frequency, respectively.
\[
(\alpha_2 D^2 + \alpha_1) u^*(y) + \alpha_2 D v^*(y) - \alpha_3 N^*(y) - \alpha_4 \theta^*(y) = 0, \tag{19}
\]
\[
d_2 D u^*(y) + (D^2 - d_2) v^*(y) - d_3 D N^*(y) - d_4 \theta^*(y) = 0, \tag{20}
\]
\[
(D^2 + d_9) N^*(y) + d_{14} \theta^*(y) = f_2(x, t) e^{-\gamma' y}, \tag{21}
\]
where
\[
D = \frac{d}{dy}, \quad d_1 = \alpha_2, \quad d_2 = \alpha_1, \quad d_3 = \beta_n, \quad d_4 = \beta_t (1 - \tau_1 (\omega) \alpha), \quad d_5 = \alpha_1 a_2 + \alpha_2 a_1, \quad d_6 = \beta_n (1 - \tau_1 (\omega) \alpha), \quad d_7 = \alpha_2 a_2 + \omega^2, \quad d_8 = \alpha_2 (1 - \tau_1 (\omega) \alpha), \quad d_9 = \alpha_2 a_2 - \alpha_2, \quad d_{10} = \alpha_2 a_2 + \omega^2, \quad d_{11} = \frac{\beta_0}{\tau_1}, \quad d_{12} = \alpha_2 a_2 + \omega^2, \quad d_{13} = \alpha_2 a_2 + \omega^2, \quad d_{14} = \beta_0 \frac{t}{t_0} \left(\frac{-x^2}{r^2} - \frac{t}{t_0}\right). \]
Now, let us proceed to solve the nonhomogeneous coupled differential equations by an eigenvalue approach. Eqs. (18)-(21) can be written in a vector-matrix differential equation as follows

$$ \frac{d\phi}{dy} = \lambda \phi - g \ e^{-\gamma y}, \quad (22) $$

where

$$ \phi = \begin{bmatrix} u & v & N & \theta & du \ dy & dv \ dy & dN \ dy & d\phi \ dy \end{bmatrix} $$

$$ \lambda = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ b_{11} & 0 & b_{12} & b_{13} & b_{14} & b_{15} & d_1 & b_{16} & b_{17} \\ 0 & b_{15} & 0 & b_{17} & 0 & b_{18} & b_{19} & 0 & 0 \\ 0 & b_{16} & b_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{20} & 0 & b_{21} & b_{22} & 0 & b_{23} & 0 & 0 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_1(x, t) \\ f_2(x, t) \end{bmatrix} $$

with

$$ b_{11} = -\frac{d_1}{d_2}, \quad b_{12} = \frac{d_3}{d_2}, \quad b_{13} = \frac{d_4}{d_2}, \quad b_{14} = \frac{d_5}{d_2}, \quad b_{15} = d_6, \quad b_{16} = d_7, \quad b_{17} = d_8, \quad b_{18} = d_9, \quad b_{19} = d_{10}, \quad b_{20} = d_{12}, \quad b_{24} = -d_7, \quad b_{25} = d_6, \quad b_{26} = -d_5, \quad b_{27} = d_4, \quad b_{28} = -d_3, \quad b_{29} = d_2. $$

Then, the characteristic equation of the matrix $\lambda$ is expressed as

$$ \xi^8 + A \xi^6 + B \xi^4 + C \xi^2 + D = 0, \quad (23) $$

where

$$ A = -b_{11} - b_{15} - b_{14}b_{17} - b_{19} - b_{22} - b_{14}b_{23}, $$

$$ B = b_{14}b_{15} + b_{11}b_{19} + b_{15}b_{10} + b_{14}b_{17}b_{19} - b_{13}b_{20} - b_{14}b_{16}b_{20} + b_{11}b_{22} + b_{15}b_{22} + b_{14}b_{17}b_{22} + b_{16}b_{22} + b_{14}b_{16}b_{23} + b_{15}b_{17}b_{23} + b_{16}b_{19}b_{23} - b_{21}b_{24} - b_{18}b_{23}b_{24}, $$

$$ C = -b_{14}b_{17}b_{19} - b_{13}b_{15}b_{20} + b_{13}b_{15}b_{20} + b_{14}b_{16}b_{19}b_{20} - b_{14}b_{15}b_{19}b_{20} - b_{14}b_{17}b_{19}b_{22} + b_{11}b_{19}b_{22} - b_{14}b_{17}b_{19}b_{22} - b_{11}b_{14}b_{19}b_{23} + b_{13}b_{15}b_{19}b_{23} - b_{12}b_{20}b_{24} - b_{14}b_{18}b_{20}b_{24} + b_{11}b_{21}b_{24} + b_{14}b_{17}b_{23}b_{24} - b_{12}b_{12}b_{22}b_{24} + b_{11}b_{18}b_{23}b_{24}, $$

$$ D = -b_{13}b_{15}b_{19}b_{20} + b_{14}b_{15}b_{19}b_{20} + b_{12}b_{13}b_{20}b_{24} - b_{11}b_{15}b_{21}b_{24}. $$

The roots of the characteristic Eq. (21) which are also the eigenvalues of matrix $\lambda$ are of the form $\pm \xi_1, \pm \xi_2, \pm \xi_3$ and $\pm \xi_4$. The eigenvector $Y = [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8]$ corresponding to eigenvalue $\xi$ can be calculated as

$$ Y_1 = -\left( (b_{12}b_{13}b_{24} - \xi b_{10}b_{24}(b_{11} - \xi^2)) + (b_{13} - \xi b_{16}(b_{11} - \xi^2))(b_{19} - \xi^2) \right), $$

$$ Y_2 = -\left( (b_{19} - \xi^2)(\xi^2b_{14}b_{17} - \xi b_{16}b_{11} + \xi^2b_{18} - b_{13}b_{15} - \xi^2b_{15} - \xi^2b_{16} + \xi^4) \right), $$

$$ Y_3 = b_{24}(\xi^2b_{12}b_{14} - (b_{11} - \xi^2)(b_{15} - \xi^2)). $$

$$ Y_4 = \frac{b_{14}b_{17}b_{19}b_{20} - \xi b_{14}b_{16}b_{20} + \xi^2b_{14}b_{24} + \xi^2b_{13}b_{20} - \xi^2b_{13}b_{19}b_{20} + \xi^2b_{16}b_{19}b_{20} + \xi^2b_{16}b_{19}b_{24} - \xi^2b_{15}b_{20}b_{24} - \xi^2b_{15}b_{19}b_{23} - b_{13}b_{15}b_{19}b_{23} - b_{12}b_{20}b_{24} - b_{14}b_{18}b_{20}b_{24} + b_{11}b_{21}b_{24} + b_{14}b_{17}b_{23}b_{24} - b_{12}b_{12}b_{22}b_{24} + b_{11}b_{18}b_{23}b_{24}, $$

$$ Y_6 = \xi Y_5, \quad Y_7 = \xi Y_6, \quad Y_8 = \xi Y_7. $$

The solution of Eq. (22) has the following form

$$ \phi = \sum_{i=1}^{4} B_i Y_i e^{-\xi_i}, \quad \sum_{i=1}^{4} B_i Y_i e^{-\xi_i} + \sum_{i=1}^{4} B_i Y_i e^{-\xi_i}, \quad g_2 e^{-\gamma y}. \quad (24) $$
where $B_i$, ($i = 1, 2, \ldots, 8$) are constants to be determined by the boundary conditions of the problem and

$$
g^* = [g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8]^T,
$$

$$
g = -\left( b_20(y^2 - b_{15}) + y^2(b_{23}b_{17}) \right) \left( y'^2b_{14}(b_{12}b_{24} + b_{13}(y^2 - b_{13})) + \left( y^2b_{13}b_{14} - (y^2 - b_{15})(y^2 - b_{11}) \right) \right).
$$

$$
g_3 = \frac{1}{\theta}(b_{20}(y^2 - b_{15}) + y^2b_{23}b_{17}) \left( y'^2b_{13}b_{14} + y^2b_{20}(y^2 - b_{13}) \right) - f_1(x, t) \left( y'^2b_{14}b_{14} - (y^2 - b_{15})(y^2 - b_{11}) \right) f_1(x, t).
$$

$$
g_4 = \left( \frac{y^2 - b_{14}}{b_{24}} \right) f_1(x, t).
$$

$$
g_5 = -y'g_5, \quad g_6 = -y'g_6, \quad g_7 = -y'g_7, \quad g_8 = -y'g_8.
$$

The general solution of the field variables can be written as

$$
\bar{u} = \sum_{i=1}^{4} B_i U_i e^{-\xi \gamma} + \sum_{i=1}^{4} B_{i+4} U_{i+4} e^{\xi \gamma} + g_1 e^{-\gamma \gamma}, \quad (25)
$$

$$
\bar{v} = \sum_{i=1}^{4} B_i V_i e^{-\xi \gamma} + \sum_{i=1}^{4} B_{i+4} V_{i+4} e^{\xi \gamma} + g_2 e^{-\gamma \gamma}, \quad (26)
$$

$$
\bar{N} = \sum_{i=1}^{4} B_i N_i e^{-\xi \gamma} + \sum_{i=1}^{4} B_{i+4} N_{i+4} e^{\xi \gamma} + g_3 e^{-\gamma \gamma}, \quad (27)
$$

$$
\bar{\theta} = \sum_{i=1}^{4} B_i \theta_i e^{-\xi \gamma} + \sum_{i=1}^{4} B_{i+4} \theta_{i+4} e^{\xi \gamma} + g_4 e^{-\gamma \gamma}, \quad (28)
$$

and

$$
\bar{u}_{yy} = \sum_{i=1}^{4} B_i (-\xi V_i + a_3 a U_i - \beta_{11} N_i - \beta_{12} (1 - \tau_{11} \omega) \theta_i) e^{-\xi \gamma} + \sum_{i=1}^{4} B_{i+4} (-\xi V_{i+4} + a_3 a U_{i+4} - \beta_{11} N_{i+4} - \beta_{12} (1 - \tau_{11} \omega) \theta_{i+4}) e^{\xi \gamma} - \left( y'g_2 - a_3 a g_1 + \beta_{11} g_3 + \beta_{12} (1 - \tau_{11} \omega) g_4 \right) e^{-\gamma \gamma}.
$$

$$
\bar{u}_{xy} = \sum_{i=1}^{4} B_i (-a_2 \xi V_i + a_2 a U_i) e^{-\xi \gamma} + \sum_{i=1}^{4} B_{i+4} (-a_2 \xi V_{i+4} + a_2 a U_{i+4}) e^{\xi \gamma} - \left( -a_2 y'g_1 + a_2 a g_2 \right) e^{-\gamma \gamma}.
$$

After applying the boundary conditions of the problem as mentioned above, we have the following set of equations:

$$
\sum_{i=1}^{4} B_i (-\xi V_i + a_3 a U_i - \beta_{11} N_i - \beta_{12} (1 - \tau_{11} \omega) \theta_i) = (y'g_2 - a_3 a g_1 + \beta_{11} g_3 + \beta_{12} (1 - \tau_{11} \omega) g_4).
$$

$$
\sum_{i=1}^{4} B_i (-a_2 \xi V_i + a_2 a U_i) = a_2 (y'g_1 - a_2 a g_2).
$$

$$
\sum_{i=1}^{4} B_i (-\xi V_i + a_3 a U_i - \beta_{11} N_i - \beta_{12} (1 - \tau_{11} \omega) \theta_i) = (y'g_2 + \frac{q t^2 e^{-1/g}}{16c t^2}).
$$
where \( B_i \) (i = 1, 2, 3, 4) are constants based on the boundary conditions of the problem, which can be determined by:

\[
\]

Therefore,

\[
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4
\end{bmatrix} =
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{bmatrix}^{-1}
\begin{bmatrix}
\gamma'g_2 - a_2\gamma g_3 + \gamma g_4 \\
\alpha_2\gamma'g_2 - a_2\gamma g_3 + \gamma g_4 \\
\gamma'g_2 + \frac{q_d}{16\pi c}e^{(-\gamma t_p)}
\end{bmatrix},
\]

in which

\[
M_{1p} = \sum_{\mu=1}^4(-\xi_p V_p + \alpha_2 a U_p - \beta_n N_p - \beta_n (1 - \tau / \omega) \theta_p),
\]

\[
M_{2p} = \sum_{\mu=1}^4(-a_2 \xi_p U_p + \alpha_2 a V_p),
\]

\[
M_{3p} = \sum_{\mu=1}^4(-D_n \xi_p N_p - s_0 N_p),
\]

\[
M_{4p} = \sum_{\mu=1}^4(-\xi_p \theta_p).
\]

Hence, we obtain the solution of each variable.

4. Numerical results and discussions

To evaluate with the numerical examples, we consider for the computational purpose the silicon (Si) material. The thermoelastic properties of such material are (Alzahrani and Abbas 2018)

\[
\begin{align*}
E &= 2.33 \text{ (eV)}, & E_g &= 1.11 \text{ (eV)}, & \rho &= 3 \times 10^{-2} \text{ (ms)}.
\end{align*}
\]

\[
\begin{align*}
\lambda &= 3.64 \times 10^{10} \text{ (Nm)}, & \mu &= 5.64 \times 10^{10} \text{ (Nm)},
\end{align*}
\]

\[
\begin{align*}
\rho &= 2330 \text{ (kg m^{-3})}, & \tau &= 5 \times 10^{-6} \text{ (s)}, & a &= 3 \times 10^{-6} \text{ (K^{-1})},
\end{align*}
\]

\[
\begin{align*}
c_e &= 695 \text{ (Jkg^{-1}K^{-1})}, & T_0 &= 300 \text{ (K)}, & K &= 1.7 \times 10^2 \text{ (Wm^{-1}K)}, & \omega &= 0.1 + 0.1 i,
\end{align*}
\]

\[
\begin{align*}
\alpha &= 0.5, & q_0 &= 0.1.
\end{align*}
\]

Based on the data set, the following graphs represent the numerically computed physical quantities at different values of the distance \( y \). Numerical computations are carried out for the displacement, distribution of the temperature, the density of carriers along the \( y \)-axis for the two-dimensional isotropic and homogenous medium in context of the coupled photo-thermo-elastic conditions.

![Fig. 1 Variation of temperature against \( y \) for different values of \( t_p \)](image-url)
As shown graphically in Figures 1-4, the distributions with different values of characteristic time of heat pulse $t_p$ i.e., $t_p = 0.1$, $0.2$ and $0.4$. The solid line refers to $t_p = 0.1$, while the dashed line shows to the time $t_p = 0.2$ and the dotted line refers to the time $t_p = 0.4$. Fig. 1 shows the variation in temperature distribution and it starts with its maximum value and gradually decreases with the distance $y$. Variation in carrier density is shown in the Fig. 2 and it also starts
with its peak value and the graph comes down with the increasing distance $y$ with a range of $0 \leq y \leq 10$. In Fig. 3, the variation in vertical displacement is shown with a wide range of $0 \leq y \leq 15$, initially graph goes up to its maximum value and after reaching its peak point it decreases to its minimum value of zero. Fig. 4 displays the distribution of the horizontal component with the same range of $0 \leq y \leq 15$; it starts with its maximum value and decreases with the increasing distance.
Graphs 5-8 show the variation with different values of thermal source for time $t = 0.05$, 0.1 and 0.2. The solid line refers to the time $t = 0.05$, while the dashed line refers to the time $t = 0.1$ and the dotted line refers to the time $t = 0.2$. Fig. 5 displays the distribution of the values of the temperature $T$ with respect to $y$-axis for different values of time $t$ with respect to wide range $0 \leq y \leq 10$. It is noted that the temperature starts by its maximum value at the first end of the strip $y = 0$ and gradually decreases with increasing distance up to zero. Figure 6 shows the distribution of the values of the carrier density with respect to $y$-axis for different values of time $t$ with the same range. It also starts with its maximum value and decreases with increasing distance $y$. Figure 7 shows the distribution of the values of the vertical displacement component $v$ with respect to $y$-axis for different times $t$ with wide range of $0 \leq y \leq 15$. It is noted that the vertical displacement first increases and goes to its peak and then decreases with the increasing distance up to zero. Figure 8 shows the distribution of the values of the displacement component $u$ with respect to $y$-axis for different times $t$, it starts with the maximum value and the decreases with a range of $0 \leq y \leq 15$.

5. Conclusions

According to the proceeding results, the time parameters $t_p$ and $t$ are having significant effects on distribution function of each variable. Based upon Eigen value approach, an analytical solution is analyzed for thermoelastic problem in semiconductor. Based on the graphical representation, it can be concluded that the characteristic time of pulse is having a decreasing effect on each variable, while the temporal variable is directly proportional to the amplitudes of each variable. Values of all physical quantities converge to zero with the increase in the distance $y$.

References

Photo-thermo-elastic interaction in a semiconductor material with two relaxation times... 51


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